I. A. (3) Randomized Complete Block Design
   (ball types - factor, golfers - blocks)
B. (1) Multiple Regression
   (dep. var. = emission of nitrous oxide
    indep. vars. = humidity, temperature
C. (2) Completely Randomized Design

II. (1) (a) \( \text{SSE (} X_i, X_j \text{)} \leq \text{SSE (} X_i \text{)} \)
(b) \( \text{SSR (} X_i, X_j \text{)} \geq \text{SSR (} X_i \text{)} \)
(c) \( \text{SST (} X_i, X_j \text{)} = \text{SST (} X_i \text{)} \)

(2) It measures the proportion of variability in Y that is accounted for by the regression
    of Y on \( X_i \) and \( X_j \).
(3) \( \text{SST (} X_i, X_j \text{)} - \text{SSE (} X_i, X_j \text{)} = \text{SSR (} X_i, X_j \text{)} \)

III (a) | Source | df | SS   | MS   | F   |
--------|-------|----|------|-----|-----|
       | Brand | 3  | 427915.25 | 142,638.4 | 60.24 |
       | Error | 12 | 28,412.5  | 2,367.7   |      |
       | Total | 15 | 456,327.75|       |     |

(b) We reject \( H_0 \) at \( \alpha = 0.05 \) if \( F > f_{3,12} (0.05) = 3.49 \)
Since 60.24 > 3.49 we reject \( H_0 \) and conclude
that there are differences among battery types.
\( R_p = r_p \sqrt{\frac{MSE}{n}} = r_p \sqrt{\frac{2367.7}{4}} = r_p (24.33) \)

\[ R_2 = 3.082 (24.33) = 74.99 \]
\[ R_3 = 3.225 (24.33) = 78.46 \]
\[ R_4 = 3.313 (24.33) = 80.61 \]

\[
\begin{array}{cccc}
B & A & D & C \\
860.5 & 570.75 & 496.25 & 433.0 \\
\end{array}
\]

<table>
<thead>
<tr>
<th>Comparison</th>
<th>( p )</th>
<th>( R_p )</th>
<th>Actual Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>B vs C</td>
<td>4</td>
<td>80.61</td>
<td>860.5 - 433 = 427.5 ✓</td>
</tr>
<tr>
<td>B vs D</td>
<td>3</td>
<td>78.46</td>
<td>860.5 - 496.25 = 364.25 ✓</td>
</tr>
<tr>
<td>B vs C</td>
<td>2</td>
<td>74.99</td>
<td>860.5 - 570.75 = 289.75 ✓</td>
</tr>
<tr>
<td>A vs C</td>
<td>3</td>
<td>78.46</td>
<td>570.75 - 433 = 137.75 ✓</td>
</tr>
<tr>
<td>A vs D</td>
<td>2</td>
<td>74.99</td>
<td>570.75 - 496.25 = 74.5 X</td>
</tr>
<tr>
<td>D vs C</td>
<td>2</td>
<td>74.99</td>
<td>496.25 - 433.0 = 63.25 X</td>
</tr>
</tbody>
</table>

| B | A | D | C |

**Lifetime per dollar is significantly higher for type B than all others. Also, type A has longer lifetime/dollar than type C.**
IV. (a) \[10 \times 3\]
(b) \[1 \quad 6 \quad 1\]
(c) First element of \(X'y\) is \(\sum_{i=1}^{10} y_i = 681\)
\[
\begin{pmatrix}
1 & \cdots & 1 \\
6 & \cdots & 6 \\
1 & \cdots & 2
\end{pmatrix}
\begin{pmatrix}
70 \\
60 \\
64
\end{pmatrix}
= 
\begin{pmatrix}
? \\
?
\end{pmatrix}
\]

V. (a) \[y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}\]
\[\sum_{i=1}^{3} \alpha_i = \sum_{j=1}^{4} \beta_j = 0\]
\[\varepsilon_{ij} \sim N(0, \sigma^2)\]

(b) Employee #4 is very different from the other 4 in that he seems to have substantially more arsenic than the other 4.

(c) Why significant? — since \(p = 0.0003\) for method

Duncan's:
\[
R_p = r_p \sqrt{\frac{MSE}{b}} = r_p \sqrt{\frac{13.89}{4}} = r_p (1.8635)
\]
\[R_2 = 3.461 (1.8635) = 6.45\]
\[R_3 = 3.587 (1.8635) = 6.68\]

<table>
<thead>
<tr>
<th>Comparison</th>
<th>(p)</th>
<th>(R_p)</th>
<th>Actual Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_2 vs. M_3)</td>
<td>3</td>
<td>1.668</td>
<td>9.0 - 6.8 = 2.2</td>
</tr>
<tr>
<td>(M_2 vs. M_1)</td>
<td>2</td>
<td>1.645</td>
<td>9.0 - 7.3 = 1.7</td>
</tr>
<tr>
<td>(M_1 vs. M_3)</td>
<td>2</td>
<td>1.645</td>
<td>7.3 - 6.8 = 0.5</td>
</tr>
</tbody>
</table>

\(M_2\) significantly higher than the other two methods.