HW #5

4.24.  

a) 1) The parameter of interest is the true mean breaking strength, \( \mu \).
2) \( H_0 : \mu = 100 \)
3) \( H_1 : \mu > 100 \)
4) \( \alpha = 0.05 \)
5) \( z_0 = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \)
6) Reject \( H_0 \) if \( z_0 > z_{\alpha} \) where \( z_{0.05} = 1.65 \)
7) \( \bar{x} = 98.03, \sigma = 2 \)
    \[
    z_0 = \frac{98.03 - 100}{2 / \sqrt{9}} = -3
    \]
8) Since \(-3 < 1.65\) do not reject \( H_0 \) and conclude that the fiber would not be judged acceptable at \( \alpha = 0.05 \).

b) P-value = \( P(Z \geq -3) = 1 - \Phi (-3) = 0.9997 \)

c) For \( \alpha = 0.05 \), accept \( H_0 \) if \( \bar{x} < 100 + 1.65 \left( \frac{2}{\sqrt{9}} \right) = 101.1 \)

\[
P(\bar{x} \leq 101.1 \text{ when } \mu = 104) = P \left( Z \leq \frac{101.1 - 104}{2 / \sqrt{9}} \right) = P(Z \leq -4.35) = \Phi(-4.35) \approx 0
\]

The probability is 0 of accepting the null hypothesis if the true mean breaking strength is 104 psi, with a level of significance of \( \alpha = 0.05 \).

d) \( z_{\alpha/2} = z_{0.025} = 1.96 \)

\[
\bar{x} - z_{0.025} \left( \frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{0.025} \left( \frac{\sigma}{\sqrt{n}} \right)
\]

\[
98.03 - 1.96 \left( \frac{2}{\sqrt{9}} \right) \leq \mu \leq 98.03 + 1.96 \left( \frac{2}{\sqrt{9}} \right)
\]

\[
96.72 \leq \mu \leq 99.34
\]

With 95% confidence, we believe the true mean breaking strength is between 96.72 psi and 99.34 psi.

\[
4.25(3)
\]

The parameter of interest is the true mean yield, \( \mu \).
\( H_0 : \mu = 90 \)
\( H_1 : \mu \neq 90 \)
\( \alpha = 0.05 \)

\[
z_0 = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}
\]

Reject \( H_0 \) if \( z_0 < -z_{\alpha/2} \) where \( -z_{0.025} = -1.96 \) or \( z_0 > z_{\alpha/2} \) where \( z_{0.025} = 1.96 \)

\[
\bar{x} = 90.48, \sigma = 3
\]

\[
\therefore z_0 = \frac{90.48 - 90}{3 / \sqrt{5}} = 0.3 \leq 0 \quad \therefore \quad \text{accept } H_0
\]

8) Since \(-1.96 < 0.36 < 1.96\) do not reject \( H_0 \) and conclude the yield is not significantly different from 90% at \( \alpha = 0.05 \).

b) P-value = \( 2[1 - \Phi(0.36)] = 2[1 - 0.64058] = 0.7188 \)

c) \[
n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.025} + z_{0.05})^2 \sigma^2}{(1.96 + 1.65)^2 \sigma^2} = \frac{(196 + 165)^2 \sigma^2}{(5)^2} = 4.69
\]

\[
n \approx 5.
\]
d) $\beta = \Phi\left(\frac{z_{0.025} + 90.48 - 92}{3/\sqrt{5}}\right) - \Phi\left(-\frac{z_{0.025} + 90.48 - 92}{3/\sqrt{5}}\right)$

$= \Phi(1.96 + 1.133) - \Phi(-1.96 + 1.133)$

$= \Phi(0.83) - \Phi(-3.09)$

$= 0.7967 - 0.001$

$= 0.7957.$

e) For $\alpha = 0.05, z_{0.025} \approx 1.96$

$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$

$90.48 - 1.96\left(\frac{3}{\sqrt{5}}\right) \leq \mu \leq 90.48 + 1.96\left(\frac{3}{\sqrt{5}}\right)$

$87.85 \leq \mu \leq 93.11$

With 95% confidence, we believe the true mean yield of the chemical process is between 87.85% and 93.11%.

4-26.

a) 1) The parameter of interest is the true mean hole diameter, $\mu$.

2) $H_0 : \mu = 1.75$

3) $H_1 : \mu \neq 1.75$

4) $\alpha = 0.01$

5) $z_0 = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject $H_0$ if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\bar{x} = 1.749, \sigma = 0.02$

$z_0 = \frac{1.749 - 1.75}{0.02/\sqrt{10}} = -0.158$

8) Since $-2.58 < -0.158 < 2.58$, do not reject the null hypothesis and conclude the true mean hole diameter is not significantly different from 1.75 in. at $\alpha = 0.01$.

b) P-value $= 2[1 - \Phi (0.158)] = 2[1 - 0.5628] = 0.8744$

c) Set $\beta = 1 - 0.90 = 0.10$

$n = \frac{(z_{\alpha/2} + z_p)^2 \sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2 \sigma^2}{(1.755 - 1.75)^2} \geq \frac{(2.58 + 1.29)^2 (0.02)^2}{(0.005)^2} = 239.63,$

$n \geq 240.$

d) $\beta = \Phi\left(\frac{z_{0.005} + 1.750 - 1.755}{0.02 / \sqrt{10}}\right) - \Phi\left(-\frac{z_{0.005} + 1.750 - 1.755}{0.02 / \sqrt{10}}\right)$

$\approx \Phi(2.58 + 0.79) - \Phi(-2.58 + 0.79)$

$= \Phi(3.37) - \Phi(-3.37)$

$= 0.9633 - 0.00038$

$\beta \geq 0.9637.$

e) For $\alpha = 0.01, z_{0.025} = z_{0.005} = 2.58$

$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$

$1.749 - 2.58\left(\frac{0.02}{\sqrt{10}}\right) \leq \mu \leq 1.749 + 2.58\left(\frac{0.02}{\sqrt{10}}\right)$

$1.733 \leq \mu \leq 1.765$
The confidence interval constructed contains the value 1.75, thus the true mean hole diameter could possibly be 1.75 in. using a 99% level of confidence. Since a two-sided 99% confidence interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$, the conclusions necessarily must be consistent.

4-27

a) 1) The parameter of interest is the true mean piston ring diameter, $\mu$.
2) $H_0: \mu = 74.035$
3) $H_1: \mu \neq 74.035$
4) $\alpha = 0.01$
5) $z_0 = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$
6) Reject $H_0$ if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$
7) $\overline{x} = 74.036$, $\sigma = 0.001$
   $$z_0 = \frac{74.036 - 74.035}{0.001 / \sqrt{15}} = 3.87$$
8) Since $3.87 > 2.58$, reject the null hypothesis and conclude the true mean piston ring diameter is not 74.035 mm.

b) P-value = $2[1 - \Phi (3.87)] = 2[1 - 0.99995] = 0.0001$
   Again since the P-value is less than the level of significance, $\alpha$, we would reject the null hypothesis.

c) For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$
   $$\overline{x} - z_{0.005} \left( \frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \overline{x} + z_{0.005} \left( \frac{\sigma}{\sqrt{n}} \right)$$
   $$74.036 - 2.58 \left( \frac{0.001}{\sqrt{15}} \right) \leq \mu \leq 74.036 + 2.58 \left( \frac{0.001}{\sqrt{15}} \right)$$
   $$74.035 \leq \mu \leq 74.037$$
   With 99% confidence, we believe the true mean piston ring diameter is between 74.035 and 74.037 mm.

d) For $\alpha = 0.05$, $z_{\alpha} = z_{0.05} = 1.65$
   $$\overline{x} - z_{0.05} \frac{\sigma}{\sqrt{n}} \leq \mu$$
   $$74.036 - 1.65 \frac{0.001}{\sqrt{15}} \leq \mu$$
   $$74.0356 \leq \mu$$
   With 95% confidence, the true mean piston ring diameter is at least 74.0356 mm.

4-28

a) 1) The parameter of interest is the true mean life, $\mu$.
2) $H_0: \mu = 540$
3) $H_1: \mu > 540$
4) $\alpha = 0.05$
5) $z_0 = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$
6) Reject $H_0$ if $z_0 > z_{\alpha}$ where $z_{0.05} = 1.65$
7) $\overline{x} = 551.33$, $\sigma = 20$
   $$z_0 = \frac{551.33 - 540}{20 / \sqrt{15}} = 2.19$$
8) Since $2.19 > 1.65$, reject the null hypothesis and conclude there is sufficient evidence to support the claim the life exceeds 540 hrs at $\alpha = 0.05$.

b) P-value = $P(Z > 2.19) = 1 - P(Z \leq 2.19) = 1 - \Phi (2.19) = 0.0143$. 

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59321.04 \leq \mu \leq 63662.96

With 95% confidence, we believe the true mean tire life is between 59321.04 km and 63662.96 km.

4-34. In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.  
1) The parameter of interest is the true Izod impact strength, \( \mu \).  
2) \( H_0: \mu = 1.0 \)  
3) \( H_1: \mu > 1.0 \)  
4) \( \alpha = 0.01 \)  
5) \( t_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}} \)  
6) Reject \( H_0 \) if \( t_0 > t_{0.01, \nu} \) where \( t_{0.01, 19} = 2.539 \)  
7) \( \bar{x} = 1.121 \quad s = 0.328 \quad n = 20 \)  
   \[
t_0 = \frac{1.121 - 1.0}{0.328 / \sqrt{20}} = 1.65
   \]
8) Since 1.65 < 2.539, do not reject the null hypothesis and conclude there is not sufficient evidence to indicate that the true Izod impact strength is greater than 1.0 ft-lb/in at \( \alpha = 0.01 \).

4-35. In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.  

a) 1) The parameter of interest is the mean life in hours, \( \mu \).  
2) \( H_0: \mu = 5500 \)  
3) \( H_1: \mu > 5500 \)  
4) \( \alpha = 0.05 \)  
5) \( t_0 = \frac{\bar{x} - \mu}{s / \sqrt{n}} \)  
6) Reject \( H_0 \) if \( t_0 > t_{0.05, \nu} \) where \( t_{0.05, 14} = 1.761 \)  
7) \( \bar{x} = 5625.1 \quad s = 226.1 \quad n = 15 \)  
   \[
t_0 = \frac{5625.1 - 5500}{226.1 / \sqrt{15}} = 2.14
   \]
8) Since 2.14 > 1.761, reject the null hypothesis and conclude there is sufficient evidence to indicate that the amount of current necessary is not 300 microamps at \( \alpha = 0.05 \).

b) P-value = \( P(t > 2.14) \): for degrees of freedom of 14 we obtain  
0.025 < P-value < 0.05;

c) \( \bar{x} - t_{0.05,14} \left( \frac{s}{\sqrt{n}} \right) \leq \mu \)  
\[
5625.1 - 1.761 \left( \frac{226.1}{\sqrt{15}} \right) \leq \mu
\]
5522.3 \leq \mu

We are 95% confident that the true mean life is at least 5522.3. Therefore, the confidence interval also indicates that the null hypothesis is rejected since 5500 is not contained in this interval.
The normality assumption appears to be satisfied. This is evident by the fact that the data fall along a straight line.

4.36:

a) 1) The parameter of interest is the true mean level of polyunsaturated fatty acid, \( \mu \).
   2) \( H_0 : \mu = 17 \)
   3) \( H_1 : \mu \neq 17 \)
   4) \( \alpha = 0.01 \)
   5) \( t_0 = \frac{\overline{x} - \mu}{s / \sqrt{n}} \)
   6) Reject \( H_0 \) if \( t_0 < -t_{0.005,5} \) where \( -t_{0.005,5} = -4.032 \) or \( t > t_{0.005,5} \) where \( t_{0.005,5} = 4.032 \)
   7) \( \overline{x} = 16.98 \) \( s = 0.319 \) \( n = 6 \)

\[
t_0 = \frac{16.98 - 17}{0.319 / \sqrt{6}} = -0.154
\]

8) Since \( -4.032 < -0.154 < 4.032 \), do not reject the null hypothesis and conclude the true mean level is not significantly different from 17\% at \( \alpha = 0.01 \).

b) P-value = \( 2P(t > 0.154) \): for degrees of freedom of 5 we obtain

\[
2(0.40) < \text{P-value}
\]

\[
0.80 < \text{P-value}
\]

c) Using the OC curves on Chart IIIb, with \( d = \frac{0.5}{0.319} = 1.567 \), \( n = 10 \), when \( \beta \equiv 0.1 \). Therefore, the current sample size of 6 is inadequate.

d) For \( \alpha = 0.01 \), \( t_{0.005,5} = 4.032 \)

\[
\overline{x} - t_{0.005,5} \left( \frac{s}{\sqrt{n}} \right) \leq \mu \leq \overline{x} + t_{0.005,5} \left( \frac{s}{\sqrt{n}} \right)
\]

\[
16.98 - 4.032 \left( \frac{0.319}{\sqrt{6}} \right) \leq \mu \leq 16.98 + 4.032 \left( \frac{0.319}{\sqrt{6}} \right)
\]

\[
16.455 \leq \mu \leq 17.505
\]

With 99\% confidence, we believe the true mean level of polyunsaturated fatty acid is between 16.455\% and 17.505\%.

4.37. a) According to the normal probability plot, the data appear to follow a normal distribution. This is evident by the fact that the data fall along a straight line.
1) The parameter of interest is the true standard deviation of the diameter, \( \sigma \). However, the answer can be found by performing a hypothesis test on \( \sigma^2 \).
2) \( H_0: \sigma^2 = 0.0004 \)
3) \( H_1: \sigma^2 > 0.0004 \)
4) \( \alpha = 0.01 \)
5) \( \chi_0^2 = \frac{(n-1)s^2}{\sigma^2} \)
6) Reject \( H_0 \) if \( \chi_0^2 > \chi_{\alpha, n-1}^2 \) where \( \chi_{0.05, 14}^2 = 23.685 \)
7) \( n = 15, \; s = 0.016 \)
\[ \chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.016)^2}{0.0004} = 8.96 \]
8) Since \( 8.96 < 23.685 \) do not reject \( H_0 \) and conclude there is insufficient evidence to indicate the true standard deviation of the diameter exceeds 0.02 at \( \alpha = 0.05 \).

b) \( P\text{-value} = P(\chi^2 > 8.96) \) for 14 degrees of freedom:
\[ 0.5 < P\text{-value} < 0.9 \]

c) 95% lower confidence interval on \( \sigma^2 \):
For \( \alpha = 0.05 \) and \( n = 15, \chi_{\alpha, n-1}^2 = \chi_{0.05, 14}^2 = 23.68 \)
\[ \frac{14(0.016)^2}{23.68} < \sigma^2 \]
\[ 0.00015 < \sigma^2 \]

With 95% confidence, we believe the true variance of the hole diameter is greater than 0.00015 mm\(^2\).
With 95% confidence, we believe the true standard deviation of the hole diameter is greater than 0.012 mm.

\( 4-44 \)

a) In order to use \( \chi^2 \) statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true variance of the sugar content, \( \sigma^2 \).
2) \( H_0: \sigma^2 = 18 \)
3) \( H_1: \sigma^2 \neq 18 \)
4) \( \alpha = 0.05 \)
5) \( \chi_0^2 = \frac{(n-1)s^2}{\sigma^2} \)
6) Reject \( H_0 \) if \( \chi_0^2 < \chi_{1-\alpha/2, n-1}^2 \) where \( \chi_{0.975, 9} = 2.70 \) or \( \chi_0^2 > \chi_{\alpha, 2, n-1}^2 \) where \( \chi_{0.025, 9} = 19.02 \)
7) \( n = 10, \; s^2 = 16 \)
\[ \chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(16)}{18} = 8 \]
8) Since \( 2.70 < 8 < 19.02 \) do not reject \( H_0 \) and conclude the evidence indicates the true variance of the sugar content is not significantly different from 18 mg\(^2\) at \( \alpha = 0.05 \).

b) \( P\text{-value} = 2P(\chi^2 > 8) \geq 1 \) for 9 degrees of freedom

c) 95% confidence interval for \( \sigma \):
First find a confidence interval for \( \sigma^2 \):
For \( \alpha = 0.05 \) and \( n = 10, \chi_{\alpha/2, n-1}^2 = \chi_{0.025, 9} = 19.02 \) and \( \chi_{1-\alpha/2, n-1}^2 = \chi_{0.975, 9} = 2.70 \)
\[ \frac{9(16)}{19.02} \leq \sigma^2 \leq \frac{9(16)}{2.70} \]
\[ 7.57 \leq \sigma^2 \leq 53.33 \]
\[ \sigma \leq \sqrt{7.57} \leq \sigma \leq \sqrt{53.33} \]
\[ 2.7 \leq \sigma \leq 7.3 \]
2.75 \leq \sigma \leq 7.30

With 95% confidence, we believe the true standard deviation of the sugar content is between 2.75 mg and 7.30 mg.

4-45. a) In order to use \( \chi^2 \) statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the standard deviation of tire life, \( \sigma \). However, the answer can be found by performing a hypothesis test on \( \sigma^2 \).
2) \( H_0 : \sigma^2 = 16000000 \)
3) \( H_1 : \sigma^2 > 16000000 \)
4) \( \alpha = 0.05 \)
5) \( \chi^2_0 = \frac{(n-1)s^2}{\sigma^2} \)
6) Reject \( H_0 \) if \( \chi^2_0 > \chi^2_{0.05; n-1} \) where \( \chi^2_{0.05; n-1} = 16.919 \)
7) \( n = 10 \), \( s^2 = 3035^2 \)
   \[ \chi^2_0 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(3035)^2}{16000000} = 5.181 \]
8) Since \( 5.181 < 16.919 \) do not reject \( H_0 \) and conclude there is no evidence to indicate the true standard deviation of tire life exceeds 4000 km at \( \alpha = 0.05 \).

b) P-value = \( P(\chi^2 > 5.181) \) for 9 degrees of freedom, \( 0.50 < \text{P-value} < 0.90 \).

c) 95% lower confidence interval for \( \sigma^2 \):

For \( \alpha = 0.05 \) and \( n = 10 \), \( \chi^2_{0.05; n-1} = 16.919 \)
\[
\frac{9(3035)^2}{16.919} < \sigma^2
\]
\[
4899877.36 < \sigma^2
\]

With 95% confidence, we believe the true variance of tire life is greater than 4,899,877.36 km².
With 95% confidence, we believe the true standard deviation of tire life is greater than 2213.57 km.

4-46. a) In order to use \( \chi^2 \) statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of Izod strength, \( \sigma \). However, the answer can be found by performing a hypothesis test on \( \sigma^2 \).
2) \( H_0 : \sigma^2 = 0.01 \)
3) \( H_1 : \sigma^2 \neq 0.01 \)
4) \( \alpha = 0.01 \)
5) \( \chi^2_0 = \frac{(n-1)s^2}{\sigma^2} \)
6) Reject \( H_0 \) if \( \chi^2_0 < \chi^2_{1-\alpha/2; n-1} \) where \( \chi^2_{0.995;19} = 6.84 \) or \( \chi^2_0 > \chi^2_{0.005;2, n-1} \) where \( \chi^2_{0.005;19} = 38.58 \)
7) \( n = 20 \), \( s = 0.328 \)
   \[ \chi^2_0 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.328)^2}{0.01} = 204.41 \]
8) Since \( 204.41 > 38.58 \) we would reject \( H_0 \) and conclude the true standard deviation of Izod strength is significantly different from 0.10 ft-lb/in at \( \alpha = 0.01 \).

b) P-value = \( 2P(\chi^2 > 204.41) \) for 19 degrees of freedom
   \[ P(\chi^2 > 204.41) < 0.005 \]
   \[ \text{P-value} < 0.01 \]