EXAM II Key

I. 1. independent samples
   2. (5)
   3. (d)

II (a) 1. proportions failing heat tolerance test
   2. \( H_0: p_1 = p_2 \)  \( 1 = \text{natural}, 2 = \text{synthetic} \)
   3. \( H_1: p_1 < p_2 \)
   4. \( \alpha = 0.05 \)
   5. \( Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \)

   \( \hat{p} = \frac{\text{successes}}{\text{total}} \)

   6. RR: Reject \( H_0 \) if \( Z_0 < -Z_{0.05} \), i.e., \( Z_0 < -1.64 \)
   7. \( \hat{p}_1 = \frac{34}{225} = 0.15 \) \( \hat{p}_2 = \frac{45}{225} = 0.2 \)

   \[ Z_0 = \frac{0.15 - 0.2}{\sqrt{\left( \frac{0.15 \cdot 0.85}{225} + \frac{0.25 \cdot 0.75}{225} \right)}} = \frac{-0.07}{0.358} = -0.2 \]

   8. Since -1.95 < -1.645 we reject \( H_0 \)
   and conclude that natural fiber is more heat tolerant.

   P-value = 0.025588
\[
\begin{align*}
(6) \quad n &= (2.05 \left( \frac{1.7 + 2.4 \times 0.3 + 7.0}{2} \right) + 2.1 \left( \frac{1.7 (0.3) + 2.4 (7.0)}{2} \right)) \frac{1.7 - 2.4}{2} \\
&= \frac{1.645 (0.5709) + 1.28 (0.5688)}{107}^2 \\
&= \left( \frac{0.931 + 0.72806}{100.49} \right)^2 = \left( \frac{1.66}{100.49} \right)^2 \\
&= 5.672 \text{ or } 5.68
\end{align*}
\]

\section*{III}

\begin{align*}
\delta_1 &= 3 - 5 = -2 \\
\delta_2 &= 12 - 3 = 9 \\
\delta_3 &= 5 - 2 = 3 \\
\delta_4 &= 4 - 1 = 3 \\
\delta_5 &= 6 - 4 = 2 \\
\delta_6 &= 3 - 4 = -1
\end{align*}

\[
\delta = 3.17
\]

\[
\bar{d} \pm 1.645 \delta d = \text{i.e. } 3.17 \pm 1.645 (3.17) = 3.17 \pm 5.35
\]

\[\text{at } 2.18 \rightarrow 8.52\]

No, since this interval contains zero and also values for which \( \mu d < 0 \), i.e., \( \mu d < \mu \), we cannot conclude that lighting reduces accidents.
IV. (a) Average gas mileage for the 3 brands
(b) \( H_0: \mu_1 = \mu_2 = \mu_3 \)
(c) \( H_i: \) not all means are equal
4. \( \alpha = .01 \)
5. Test Statistic: \( F_0 = \frac{M^2\text{Treatment}}{M^2\text{Error}} = F_{a-1, a(n-1)} \) when \( H_0 \) is true
6. Rejection Region: Reject \( H_0 \) if \( F_0 > f_{.01, 2, 3} \)
   \[ \text{i.e. } F_0 > 6.93 \]
7. \( \text{SS}_{\text{total}} = \sum \frac{y_{ij}^2}{n} - \left( \sum \frac{y_{ij}}{n} \right)^2 \)
   \[ = 6112 - \frac{(300)}{15} \]
   \[ = 6112 - 6000 = 112 \]
   \( \text{SS}_{\text{Treatment}} = \sum \frac{y_{ij}^2}{n} - \left( \sum \frac{y_{ij}}{n} \right)^2 \)
   \[ = 6077.2 - 6000 = 77.2 \]
   \( \text{SS}_{\text{Error}} = 112 - 77.2 = 34.8 \)
8. \( F_0 = \frac{112/2}{34.8/12} = \frac{38.6}{2.9} = 13.31 \)
   \( \text{Since } 13.31 > 6.93 \text{ we reject } H_0. \)
   \( \text{and conclude that the average gas mileage is not the same for all 3 brands. Examination of the data indicates that Brand C has higher gas mileage than the other two brands.} \)
\[
\frac{\frac{1}{\sqrt{2^2}}}{5,1^2} f_{.175}, 7,9 = \frac{\frac{1}{\sqrt{2^2}}}{5,3^2} f_{.025}, 7,9 = \frac{1}{\sqrt{2^2}} f_{.175}, 7,9 \leq \frac{1}{\sqrt{2^2}} f_{.025}, 7,9 \leq \frac{1}{\sqrt{2^2}} \frac{16.81}{28.09} \leq \frac{\frac{1}{\sqrt{2^2}}}{28.09} (4.20) \leq \frac{1}{\sqrt{2^2}} (4.20) \leq 1.124 \leq \frac{1}{\sqrt{2^2}} \leq 2.5^1/3
\]

Since this interval includes 1, we cannot conclude that there is a difference between \( \sigma_1^2 \) and \( \sigma_2^2 \).