Instructions: This should be treated as an open-book, timed exam. You may use any materials from previous classes when taking this exam. The time limit is two hours. If you are able to ace this exam, you are more than ready for Calculus III.
Problem 1. Basic Algebra.

(a) Reduce to simplest form the expression \( e^{-2 \ln(x)} \).

(b) Solve for \( x \) if
\[ e^\pi = \sqrt{2 \ln(x) - 1} \]

(c) Find the unknowns \( x \) and \( y \) in the linear system
\[
\begin{align*}
2x + 3y &= 8 \\
3x + 2y &= 7
\end{align*}
\]
Problem 2. Differentiation.

(a) Using appropriate rules, compute the derivative $\frac{dy}{dx}$ if

$$y = e^x \frac{x}{x + 1}$$

(b) Use implicit differentiation to find $\frac{dy}{dx}$ if

$$x = \tan(y)$$

(Hint: draw a triangle to help you write the result in terms of $x$).
Problem 3. Some basic integrals. Compute the following indefinite integrals:

(a) $\int 2t\sqrt{1+t^2} dt$
(b) $\int ze^z dz$
Problem 4. Parametric Curves. A particle traces out the trajectory given by the parametric curve
\[x = t^3 - t\]
\[y = \sqrt{3} (t^2 - 1)\]

(a) Sketch the graphs of the functions \(x(t)\) and \(y(t)\).

(b) Using the results from part (a), sketch the trajectory of the particle in the \((x, y)\) plane.

(b) What is the total distance travelled by the particle during the interval \(t \in [-1, 1]\)?
Problem 5. Polar Co-ordinates. Consider the polar curve

\[ r = \sin(2\theta). \]

(a) Sketch the graph of this function for \( \theta \in [0, 2\pi] \).

(b) What is the area enclosed by the graph over the interval \( \theta \in \left[0, \frac{\pi}{2}\right] \)?
Problem 6. Avoiding Common Mistakes. Consider each of the following calculations performed by students. Identify where the student went wrong in each case, and identify what the student needed to have done instead. Then calculate the correct answer.

(a) \( \int \frac{1}{x(x + 1)} \, dx = -\frac{1}{2} \ln (x + 1) + C \)

(b) \( \int x \cos (x) \, dx = \frac{1}{2} x^2 \sin (x) + C \)

(c) \( \int (a + x) \, dx = \frac{1}{2} a^2 + \frac{1}{2} x^2 + C \)