8. Since \(2r^2 + z^2 = 1\) and \(r^2 = x^2 + y^2\), we have \(2(x^2 + y^2) + z^2 = 1\) or \(2x^2 + 2y^2 + z^2 = 1\), an ellipsoid centered at the origin with intercepts \(x = \pm \frac{1}{\sqrt{2}}, y = \pm \frac{1}{\sqrt{2}}, z = \pm 1\).

10. (a) Substituting \(x = r \cos \theta\) and \(y = r \sin \theta\), the equation \(3x + 2y + z = 6\) becomes \(3r \cos \theta + 2r \sin \theta + z = 6\) or \(z = 6 - r(3 \cos \theta + 2 \sin \theta)\).

(b) The equation \(-x^2 - y^2 + z^2 = 1\) can be written as \(-(x^2 + y^2) + z^2 = 1\) which becomes \(-r^2 + z^2 = 1\) or \(z^2 = 1 + r^2\) in cylindrical coordinates.

\[
z = r = \sqrt{x^2 + y^2}\]
is a cone that opens upward. Thus \(r \leq z \leq 2\) is the region above this cone and beneath the horizontal plane \(z = 2\). \(0 \leq \theta \leq \frac{\pi}{4}\) restricts the solid to that part of this region in the first octant.

12. The paraboloid \(z = x^2 + y^2 = r^2\) intersects the plane \(z = 4\) in the circle \(x^2 + y^2 = 4\) or \(r = 2\), so in cylindrical coordinates, \(E\) is given by \(\{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi, 0 \leq r \leq 2, r^2 \leq z \leq 4\}\). Thus

\[
\iiint_E z\,dV = \int_0^{2\pi} \int_0^2 \int_{r^2}^4 (z) r\,dz\,dr\,d\theta = \int_0^{2\pi} \int_0^2 \left[ \frac{1}{2}rz^2 \right]_{r^2}^4 \,dr\,d\theta
\]

\[
= \int_0^{2\pi} \int_0^2 (8r - \frac{1}{2}r^5) \,dr\,d\theta = \int_0^{2\pi} \int_0^2 (8r - \frac{1}{2}r^5) \,dr\,d\theta = 2\pi \left[ 4r^2 - \frac{1}{12}r^6 \right]_0^2 = 2\pi \left( 16 - \frac{16}{3} \right) = \frac{64}{3}\pi
\]

20. In cylindrical coordinates \(E\) is bounded by the planes \(z = 0, z = r \cos \theta + r \sin \theta + 5\) and the cylinders \(r = 2\) and \(r = 3\), so \(E\) is given by \(\{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi, 2 \leq r \leq 3, 0 \leq z \leq r \cos \theta + r \sin \theta + 5\}\). Thus

\[
\iiint_E \,dV = \int_0^{2\pi} \int_2^3 \int_{r \cos \theta + r \sin \theta + 5}^r (r \cos \theta) r\,dz\,dr\,d\theta = \int_0^{2\pi} \int_2^3 \left( r^2 \cos \theta \right) \left[ z \right]_{z=r \cos \theta + r \sin \theta + 5}^{z=r} \,dr\,d\theta
\]

\[
= \int_0^{2\pi} \int_2^3 \left( r^2 \cos \theta \right) (r \cos \theta + r \sin \theta + 5) \,dr\,d\theta = \int_0^{2\pi} \int_2^3 \left( r^2 \cos \theta + r \cos \theta \sin \theta \right) + 5r^2 \cos \theta \,dr\,d\theta
\]

\[
= \int_0^{2\pi} \left( \frac{1}{4}r^4 \cos \theta + \frac{3}{2}r^2 \cos \theta \right) \,dr\,d\theta
\]

\[
= \int_0^{2\pi} \left( \frac{1}{4}r^4 \cos \theta + \frac{3}{2}r^2 \cos \theta \right) \,dr\,d\theta = \left[ \frac{9}{8} \sin \theta + \frac{65}{16} \sin 2\theta + \frac{65}{8} \sin^2 \theta + \frac{95}{3} \sin \theta \right]_0^{2\pi} = \frac{65}{4}\pi
\]

28. Since density is proportional to the distance from the \(z\)-axis, we can say \(\rho(x, y, z) = K \sqrt{x^2 + y^2}\). Then

\[
m = 2\int_0^{2\pi} \int_0^a \sqrt{a^2 - r^2} K r\,dr\,d\theta = 2K \int_0^{2\pi} \int_0^a r^2 (a^2 - r^2) \,dr\,d\theta = 2K \int_0^{2\pi} \left[ \left( \frac{1}{3}a^4 \sin^{-1} \left( \frac{r}{a} \right) \right) \right]_0^a \,d\theta = 2K \int_0^{2\pi} \left[ \left( \frac{1}{3}a^4 \sin^{-1} \left( \frac{r}{a} \right) \right) \right]_0^a \,d\theta = \frac{1}{4}a^4 \pi^2 K