Name: **Solutions**

Math 2339: Quiz 9 - Gradient

(1) [6 pts]
Find the directional derivative of the function at the point given in the direction of \( v \).

\[ g(p, q) = \sin(p^2q) - p^2q, \quad (2, 1), \quad v = 2i + j \]

\[ \nabla g = \left< \frac{\partial g}{\partial p}, \frac{\partial g}{\partial q} \right> = \left< 2pq \cos(p^2q), -2pq \right> \]

\[ \nabla g|_{(2, 1)} = \left< 4 \cos(4), -4 \right> \]

\[ u = \frac{v}{|v|} = \frac{\left< 2, 1 \right>}{\sqrt{5}} \]

\[ D_u g = \nabla g|_{(2, 1)} \cdot \frac{\left< 2, 1 \right>}{\sqrt{5}} = \frac{12}{\sqrt{5}} \cos(4) - \frac{12}{\sqrt{5}} \]

(2) [4 pts]
Assume that the following contour plot comes from a function \( f(x, y) \). For each of the points labeled A and B sketch a path that follows \(-\nabla f\) all the way to the edge of the map.