Math 2339: Quiz 15 - Triple integrals

(1) [5 pts]
Set up but do not evaluate the integral of the function \( f(x, y, z) = xy \) over the volume \( E \) bound by the curves \( y = x^3 \), \( y = x \), the plane \( z = 0 \) and \( z = x + 2y \).

\[
\iiint_E \, xy \, dV
\]

(2) [5 pts]
Find the volume of the solid between the surfaces \( z = 4 - x^2 \) and \( z = 0 \), and between the planes \( y = 1 \) and \( y = 3 \).

\[
V = \int_{-2}^{2} \int_{1}^{3} \int_{0}^{4-x^2} \, dz \, dy \, dx
\]

\[
= \int_{-2}^{2} \left( 4 - x^2 - 1 \right) \, dx
\]

\[
= \int_{-2}^{2} \left( 3 - x^2 \right) \, dx
\]

\[
= \left[ 3x - \frac{x^3}{3} \right]_{-2}^{2}
\]

\[
= 2 \left( 4 - \frac{8}{3} \right) - 2 \left( -8 + \frac{8}{3} \right) = 4 \left( 8 - \frac{8}{3} \right) = \frac{64}{3}
\]