Problem Set 2

Answer Key

1. (a) Foreign currency controls and indeterminacy of the exchange rate express the behavior of the exchange rate under different international monetary arrangements.

The policy of foreign currency controls permits citizens of each country to hold only their own country’s currency over time. The value of country’s fiat money is determined by its own money market clearing.

The indeterminacy of the exchange rate represents the fact that people can hold and use the money of any country at any time. The exchange rate becomes indeterminate in this case.

(b) Both cooperative stabilization and unilateral defense of the exchange rate are arrangements used for solving the indeterminacy problem by having the exchange rate fixed.

The fixed exchange rate can be supported with the full cooperation from both countries under cooperative stabilization, while the only domestic government must keep its obligation without any support from foreign government under unilateral defense.

(c) Tobin effect is the substitution of capital for fiat money when there is an increase in anticipated inflation.

Fisher effect is the adjustment of the nominal interest rate according to anticipated inflation.

2. (a) You are given that each young person desires to hold money balances worth 18 goods. This implies that in a stationary equilibrium \( y^a - c^a_i = y^b - c^b_i = 18 \).

Since \( N^a = N^b = 100 \), this implies the world demand for money is \( N^a (y^a - c^a_i) + N^b (y^b - c^b_i) = 100(18) + 100(18) = 3600 \).

Because there are 200 total members of the initial old (across both countries) and each member of the initial old holds $3, the total supply of dollars is \( M^a = (200)(3) = 600 \). Similarly, the total supply of pounds is \( M^b = (200)(£3) = £600 \).

The world money market-clearing condition is:
\[ \nu_i^a M_i^a + \nu_i^b M_i^b = N_i^a (y^a - c_{1,t}^a) + N_i^b (y^b - c_{1,t}^b). \]

Given that the (fixed) exchange rate is \( \bar{e} = \nu_i^a / \nu_i^b \Rightarrow \nu_i^a = \bar{e} \nu_i^b \), this implies the following alternate expression for the world money market-clearing condition:

\[ \bar{e} \nu_i^b M_i^a + \nu_i^b M_i^b = N_i^a (y^a - c_{1,t}^a) + N_i^b (y^b - c_{1,t}^b). \]

We are given that the fixed exchange rate is \( \bar{e} = 2 \). Substituting the known values into the world money market-clearing condition:

\[ (2) \nu_i^b (600) + \nu_i^b (600) = 3600. \]

\[ 1800 \nu_i^b = 3600 \]

\[ \nu_i^b = 2. \]

Since \( \nu_i^a = \bar{e} \nu_i^b \), we find that \( \nu_i^a = (2)(2) = 4 \). The consumption of each old person is \( c_2 = \nu_i^a m_i^a + \nu_i^b m_i^b = (4)(3) + (2)(3) = 18 \) goods.

(b) This is an example of cooperative stabilization of the exchange rate. Each member of the initial old turns in $1 to the monetary authority of country \( a \) to acquire country \( b \) money. With 200 initial old across the two countries, the total amount of dollars turned into the monetary authority is (200)($1) = $200. Since the exchange rate is fixed at \( \bar{e} = 2 \), the monetary authority of country \( b \) must print £400 to accommodate this exchange. After the exchange, the total fiat money stock of each country will become \( M_i^a = $400 \) and \( M_i^b = £1,000 \). To determine the values of each country’s money, we use the world money market-clearing condition:

\[ \bar{e} \nu_i^b M_i^a + \nu_i^b M_i^b = N_i^a (y^a - c_{1,t}^a) + N_i^b (y^b - c_{1,t}^b). \]

\[ (2) \nu_i^b (400) + \nu_i^b (1000) = 3600. \]

\[ 1800 \nu_i^b = 3600 \]

\[ \nu_i^b = 2. \]

The value of country \( b \) money is unchanged. Since \( \nu_i^a = \bar{e} \nu_i^b \), we find that \( \nu_i^a = (2)(2) = 4 \). The value of country \( a \) money is also unchanged. This confirms that with cooperative stabilization, an exchange of one country’s money for the other’s does not affect the values of the two currencies. With each old person now holding $2 and £5, the consumption of each old person is

\[ c_2 = \nu_i^a m_i^a + \nu_i^b m_i^b = (4)(2) + (2)(5) = 18 \] goods.
Consumption of the initial old is unchanged under the policy of cooperative stabilization.

(c) This is an example of a unilateral defense of the exchange rate by country $a$. With 200 individuals across the two countries exchanging $1$ for £$2$, the total number of pounds that must be acquired by country $a$ is £$400$. The total real value of the tax is $\nu_i^b$ (£$400$). Since country $a$ can only tax its own citizens, each old person of country $a$ is required to pay a tax of $400\nu_i^b / 100 = 4\nu_i^b$.

Since each of 200 initial old people have turned in $1$ each, the total fiat money stock of country $a$ falls to $400$. However, unlike the case of cooperative stabilization, the fiat money stock of country $b$ remains unchanged at £$600$. We use the world money market-clearing condition to determine the value of country $b$ currency:

$$\bar{e}_i \nu_i^b M_i^a + \nu_i^b M_i^b = N_i^a(y^a - e_i^a) + N_i^b(y^b - e_i^b).$$

$$2\nu_i^b (400) + \nu_i^b (600) = 3600.$$

$$1400\nu_i^b = 3600$$

$$\nu_i^b \approx 2.5714.$$

Since the exchange rate is fixed at 2, the value of country $a$ money rises to $\approx (2)(2.57) = 5.1428$. We now can determine the effect of this policy on the consumption of the initial old in each country. Each old person now possesses $2$ and £$5$. Since the old of country $b$ pay no tax, their consumption is now

$$c_i^b = \nu_i^a m_i^a + \nu_i^b m_i^b \approx (5.1428)(2) + (2.5714)(5) = 23.1426 \text{ goods},$$

which is larger than under the cooperative stabilization policy by $23.1426 - 18 = 5.1426$ goods. The old of country $b$ benefit from the tax since the values of their currency holdings rise and they pay no tax. With the value of country $b$ at approximately 2.5714, the tax paid by each member of the initial old of country $a$ is $4\nu_i^b \approx 4(2.5714) = 10.2856 \text{ goods}$. The consumption of each of the initial old of country $a$ is now

$$c_i^a = \nu_i^a m_i^a + \nu_i^b m_i^b - (\text{tax}) \approx (5.1428)(2) + (2.5714)(5) - 10.2856 = 12.857 \text{ goods}.$$

This amount is smaller than under the cooperative stabilization policy by $18 - 12.857 = 5.143$ goods. The amount gained by country $b$ citizens is equal to the amount lost by country $a$ citizens (with a bit of rounding error). Clearly, the initial old of country $b$ prefer the policy of unilateral stabilization, whereas the initial old of country $a$ do not.
3. We are given that net population growth rate is 8 percent or 0.8. We can find gross population growth rate by adding 1 to the net rate.

\[
\text{Gross population growth rate} = 1 + 0.8 = 1.08.
\]

Also, we know that the fiat money stock is fixed. This implies that \( z = 1 \). First, we calculate the gross rate of return of fiat money.

\[
\frac{\nu_{t+1}}{\nu_t} = \frac{n}{z} = \frac{1.08}{1} = 1.08.
\]

A risk-neutral lender would be willing to make a loan if the expected real interest rate is at least equal to the rate return of fiat money (the return that a person will get if she decides to hold money instead of making a loan). Therefore, the minimum expected real interest rate a lender would accept is 1.08.

Let \( r \) be the real interest rate that is paid when the loan is repaid. The expected real interest rate, \( E(r) \), will be

\[
E(r) = \pi_1 r_1 + \pi_2 r_2 = (\text{probability of repayment})(r) + (\text{probability of default})(0).
\]

\[
= 0.9r + 0.1(0) = 0.9r.
\]

As stated, this expected real interest rate must be equal to 1.08. We have that

\[
0.9r = 1.08
\]

\[
r = 1.20.
\]

At this rate of return on loans, the expected rate of return on loans and fiat money will be equal.

We know that a risk-averse individual will require a risk premium to hold the risky asset. Hence, a risk-averse person will require a real interest rate on loans in excess of 1.20. Exactly how much in excess depends on how risk averse the person is.

4. In this problem, do not forget that net rates are equal to gross rates minus 1. We know that over a two-period horizon, capital pays the real rate of return of \( X \). Hence, over a two-period horizon, any loan must offer a two-period gross nominal rate of return \( \bar{R} \) such that
\[
X = \frac{\bar{R}}{1 + \frac{1}{p_t} R_{t+2}} = \bar{R} \left( \frac{V_{t+2}}{V_{t+1}} \right) \left( \frac{V_{t+1}}{V_t} \right) = \bar{R} \left( \frac{n}{z} \right) \left( \frac{n}{z} \right) = \bar{R} \left( \frac{n}{z} \right)^2.
\]

(This is from equations (6.10) and (6.11) in the text. See also equations (6.5) and (6.6).)

This implies that the two-period gross nominal interest rate is

\[
\bar{R} = X \left( \frac{z}{n} \right)^2.
\]

The net nominal two-period interest rate is \(\bar{R} - 1\). Furthermore, in this problem, the one-period gross real interest rate is \(r = x = \sqrt{X} = \sqrt{1.44} = 1.2\) (120\%). The net real interest rate is \(1.2 - 1 = 0.2\) (20\%). We have that

(a) The two-period net nominal interest rate is

\[
\bar{R} - 1 = X \left( \frac{z}{n} \right)^2 - 1 = (1.44) \left( \frac{1.2}{1.1} \right)^2 - 1 \approx 0.7137 = 71.37\%.
\]

From equation (6.11), the one-period net nominal interest rate is

\[
R - 1 = x \left( \frac{z}{n} \right) - 1 = (1.2) \left( \frac{1.2}{1.1} \right) - 1 \approx 0.3091 = 30.91\%.
\]

(b) The two-period net real interest rate is

\[
X - 1 = 1.44 - 1 = 0.44 = 44\%.
\]

The one-period net real interest rate is

\[
x - 1 = \sqrt{X} - 1 = \sqrt{1.44} - 1 = 1.2 - 1 = 0.2 = 20\%.
\]

(c) Over two periods, the net inflation rate is
Here we are making use of equation (6.9) of the text. Over one period, the net inflation rate is
\[
\frac{p_{t+2}}{p_t} - 1 = \left( \frac{p_{t+2}}{p_{t+1}} \right) \left( \frac{p_{t+1}}{p_t} \right) - 1 = \left( \frac{z}{n} \right) \left( \frac{z}{n} \right) - 1 = \left( \frac{\frac{z}{n}}{1} \right)^2 - 1 = \left( \frac{\frac{1.2}{1.1}}{1.1} \right)^2 - 1 \approx 0.1901 = 19.01%. 
\]

(d) The net real rate of return on fiat money over two-periods is
\[
\frac{\nu_{t+2}}{\nu_t} - 1 = \left( \frac{\nu_{t+2}}{\nu_{t+1}} \right) \left( \frac{\nu_{t+1}}{\nu_t} \right) - 1 = \left( \frac{\frac{n}{z}}{n} \right) \left( \frac{n}{\frac{n}{z}} \right) - 1 = \left( \frac{\frac{n}{z}}{1} \right)^2 - 1 = \left( \frac{\frac{1.1}{1.2}}{1.2} \right)^2 - 1 \approx -0.1597 = -15.97%. 
\]

Over one period, the net real rate of return on money is
\[
\frac{\nu_{t+1}}{\nu_t} - 1 = \left( \frac{n}{z} \right) - 1 = \left( \frac{1.1}{1.2} \right) - 1 \approx -0.0833 = -8.33%. 
\]