1. Give a short explanation of the following terms.

(a) Fiat money

Fiat money is a nearly costlessly produced commodity that can be costlessly stored, costlessly exchanged and cannot be produced or counterfeited by anyone else but the government.

(b) Feasible set

The set of possible allocations that all generation can attain, given the availability of resources in the economy at the particular time.

(c) The rate of return on fiat money

Units of good that can be obtained in period $t+1$ if one unit of good is sold for money in period $t$.

(d) Inside money

Money issued by private intermediaries.

(e) Real interest rate

The number of goods paid in interest for each good lent.
2. Consider an overlapping generations model with two countries: US (country a) and UK (country b). In each country the population of every generation is 100. Each young person in US and UK wants money balances worth 20 and 10 goods, respectively. The fiat money supplies are $400 in US and £200 in UK.

We are given \( N = 100, \ y^a - c^a_i = 20, \ y^b - c^b_i = 10, \ M^a_t = 400, \text{ and } M^b_t = 200. \)

(a) With foreign currency controls in effect, what is the value of a dollar \((v^a_t)\)?
Of a pound \((v^b_t)\)?

Under foreign currency controls, values of currency are determined independently, depending upon each country’s money market clearing.

\[
\begin{align*}
v^a_t &= \frac{N(y^a - c^a_i)}{M^a_t} = \frac{100(20)}{400} = 5 \\
v^b_t &= \frac{N(y^b - c^b_i)}{M^b_t} = \frac{100(10)}{200} = 5
\end{align*}
\]

(b) Find the exchange rate of British pounds per US dollar.

\[
e_t = \frac{v^a_t}{v^b_t} = \frac{5}{5} = 1
\]

It means that 1 US dollar can exchange for 1 UK pound.

(c) Given the exchange rate you found in part (b), what does it mean if US experiences an appreciation of its exchange rate at period \( t+1 \)? What factors can cause such a change?

It implies that 1 US dollar can exchange for more than 1 UK pound at period \( t+1; \frac{e_{t+1}}{e_t} > 1. \)

We know that \( \frac{e_{t+1}}{e_t} = \frac{v^a_{t+1}}{v^b_{t+1}} \frac{v^b_t}{v^a_t} = \left( \frac{v^a_{t+1}}{v^a_t} \right) \left( \frac{v^b_t}{v^b_{t+1}} \right) = \left( \frac{n^a}{n^a_{t+1}} \right) \left( \frac{z^b_{t+1}}{z^b} \right). \)

The US could experience an appreciation of its exchange rate when its population is growing or the money stock in UK is increasing.
(d) Without foreign currency controls in effect, find the value of each country’s money given the exchange rate you found in part (b).

Without foreign currency controls, we cannot determine the money market clearing of each country separately anymore. Given the exchange rate we found in part (b), we can find values of currency by setting the world money market clearing condition.

\[ \nu^a_t M^a_t + \nu^b_t M^b_t = N^a_t (y^a - c^a_t) + N^b_t (y^b - c^b_t) \]
\[ e_t \nu^b_t M^a_t + \nu^b_t M^b_t = N^a_t (y^a - c^a_t) + N^b_t (y^b - c^b_t) \]
\[ \nu^b_t [e_t M^a_t + M^b_t] = N^a_t (y^a - c^a_t) + N^b_t (y^b - c^b_t) \]
\[ \nu^b_t [400 + 200] = 100(20) + 100(10) \]
\[ \nu^b_t = \frac{3000}{600} = 5 \]
\[ \nu^a_t = e_t \nu^b_t = (1)(5) = 5 \]

(e) Suppose the exchange rate is still fixed at the same rate and UK triples its fiat money stock, whereas US prints no new money. How many goods will UK gain in seigniorage?

If UK triples its fiat money stock, \( M^b_t = 600 \), whereas the stock of fiat money in US stays the same, \( M^a_t = 400 \), the value of a pound will change.

\[ \nu^b_t [e_t M^a_t + M^b_t] = N^a_t (y^a - c^a_t) + N^b_t (y^b - c^b_t) \]
\[ \nu^b_t [400 + 600] = 100(20) + 100(10) \]
\[ \nu^b_t = \frac{3000}{1000} = 3 \]

Seigniorage = \( \nu^b_t [M_t - M_{t-1}] = (3)(600 - 200) = 1200 \) goods. Therefore, UK will gain 1200 goods from printing new money.

(f) What would be the optimal international monetary system? Explain and give the real world example.

The optimal international monetary system is to have a single currency among nations. Using the single currency reduces cost of exchange and eliminates transaction cost when trading. The European Union is an example where 15 EU countries use the euro as their sole official currency.
Consider a three-period-lived economy with growing population, \( N_t = 1.5N_{t-1} \). There are 100 initial old \((N_{t-2})\) at time \( t \). Individual is endowed with 100 goods when young, 10 goods when old and nothing when middle-aged. However, 10 goods are not enough for individual to survive that everyone wants to consume more than 10 goods in the third period of life. There are two assets in the economy; fiat money and capital. The fiat money stock is growing at the rate of 25\%. Capital is considered a risky asset. Two periods after capital is invested, it pays rates of return of 1.8, 0.7, and 0.5 with probabilities 0.6, 0.1, and 0.3, respectively.

We are given \( N_{t-2} = 100, n = 1.5, z = 1.25, y_1 = 100, y_2 = 0 \) and \( y_3 = 10 \).

(a) What are the rate of return on fiat money and the expected rate of return on capital?

**Rate of return on fiat money**, \[ \frac{\nu_{t+1}}{\nu_t} = \frac{n}{z} = \frac{1.5}{1.25} = 1.2 \]

**Expected rate of return on capital**, \[ E(X) = \pi_1X_1 + \pi_2X_2 + \pi_3X_3 \]
\[ = (0.6)(1.8) + (0.1)(0.7) + (0.3)(0.5) \]
\[ = 1.08 + 0.07 + 0.15 = 1.3 \]

(b) Why do people still want to hold fiat money although the rate of return on fiat money is lower?

People hold fiat money because it is more liquid. Fiat money changes hands more frequently and can be held for shorter periods of time.

(c) Find the budget constraints for individuals when young, middle-aged and old, and then obtain the lifetime budget constraint.

**Budget constraint at time \( t \):**
\[ c_1 + \nu_t m_t + k_t \leq y_1 \]

**\( t+1 \):**
\[ c_{2,t} \leq u_{t+1} m_t \]

**\( t+2 \):**
\[ c_{3,t} \leq Xk_t + y_3 \]

Solving for \( m_t \) at period \( t+1 \) and \( k_t \) at period \( t+2 \), replacing them at period \( t \), we should get the lifetime budget constraint as;

\[ c_1 + \left( \frac{\nu_t}{\nu_{t+1}} \right) c_2 + \left( \frac{1}{X} \right) c_3 \leq y_1 + \left( \frac{1}{X} \right) y_2 \Rightarrow c_1 + 1.04 c_2 + \frac{c_3}{1.3} \leq y_1 + \frac{y_2}{1.3} \]
(d) Suppose individual always wants to invest 20 goods in capital. Find the total output at time \( t \), \( GDP_t \).

\[
GDP_t = N_t y_1 + N_{t-2} y_2 + N_{t-2} X k_{t-2}
\]

Since we are given that \( N_{t-2} = 100 \) and \( n = 1.5 \), we can find

\[
N_t = (1.5)^2 N_{t-2} = (1.5)^2 (100) = 225
\]

Therefore, \( GDP_t = (225)(100) + (100)(10) + (100)(1.3)(20) = 26100 \) goods

4. Consider an economy with a constant population \( N = 100 \) in which people wish to hold bank checking deposits always worth 50 goods. Individual’s endowment is 100 goods in each period. The unintermediated capital per young person \( (k_t) \) is 10 goods in each period. Deposits at banks are subject to a reserve requirement \( (\gamma) \) of 10%. Banks invest the remainder of all deposits into capital. Capital becomes productive after two periods, and the gross rate of return on capital \( (x) \) is 1.1 per period. The total money stock \( (M^1)_t \) is $20,000 in every period.

We are given \( N = 100, h_t = 50, y_t = 100, k_t = 10, \gamma = 0.1, x = 1.1 \) and \( M^1_1 = $20000 \)

(a) Why do we call bank a financial intermediary?

Bank acts as a financial intermediary because it provides a service by correcting the mismatch of maturities between liquid money and illiquid capital. Bank accepts deposits with one-period return and use them to invest in capital with a two-period return. The arbitrage can be made through rate-of-return differences.

(b) How much would \((M^1)_t\) change if the monetary base were increased by 100?

We know that for a given reserve requirement, \( \Delta(M^1)_t = \Delta M_t / \gamma \).

So for this problem, \( \Delta(M^1)_t = 100 / 0.1 = 1000 \).
(c) Find the total capital stock and real GDP at time $t$.

Total capital \[ = N_t k_t + (1 - \gamma)N_t h_t \]
\[ = (100)(10) - (1 - 0.1)(100)(50) = 5500 \text{ goods}. \]

\[ GDP_t = N_t y_t + N_{t-2} X k_{t-2} + N_{t-2} (1 - \gamma) X h_{t-2} \]
\[ = (100)(100) + (100)(1.1)^2(10) + (100)(1 - 0.1)(1.1)^2(50) = 16655 \text{ goods.} \]

(d) Now assume that banks are allowed to borrow from the central bank. Explain intuitively how it will affect the total capital stock and GDP.

The central bank lending is generally equivalent to a decrease in the reserve requirement. It allows banks for more intermediation of capital. As the result, total capital increases, and hence, GDP.

(e) Suppose the central bank allows banks to borrow ($\delta$) 50 percent of required reserves ($\gamma N_t h_t$) at a gross interest rate ($\psi$) of 1.08. What is the gross real rate of return on deposits that will be offered by banks in competitive economy?

Banks now can borrow 50 percent of required reserves from the central bank. We know that for the interest rate of loan, the interest rate that banks will offer should be,

\[ r = \gamma \left( \frac{n}{z} \right) + [1 - \gamma (1 - \delta)]x - \psi \delta \gamma \]
\[ = (0.1)(1) + [1 - 0.1(1 - 0.5)](1.1) - (1.08)(0.1)(0.5) = 1.091 \]