Problem 1.11
No player plays a strictly dominated strategy with positive probability.
Strategies $T$ and $M$ for player 1 and $L$ and $R$ for player 2 survive iterated elimination of dominated strategies.

<table>
<thead>
<tr>
<th></th>
<th>$L$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>2, 0</td>
<td>4, 2</td>
</tr>
<tr>
<td>$M$</td>
<td>3, 4</td>
<td>2, 3</td>
</tr>
</tbody>
</table>

Suppose player 2 plays $L$ and $R$ with probability $p$ and $1-p$, $0 \leq p \leq 1$. Then, player 1’s expected payoff from strategy $T$ is $2p + 4(1-p)$ and that from strategy $M$ is $3p + 2(1-p)$. Player 1 will randomize between these two strategies only if they yield identical expected payoffs which requires:

$$2p + 4(1-p) = 3p + 2(1-p)$$

which yields $p = \frac{2}{3}$.

Suppose player 1 plays $T$ and $M$ with probability $q$ and $1-q$, $0 \leq q \leq 1$. Player 2’s expected payoff from strategy $L$ is $4(1-q)$ and that from strategy $R$ is $2q + 3(1-q)$. Player 2 will randomize between these two strategies only if they yield identical expected payoffs which requires:

$$4(1-q) = 2q + 3(1-q)$$

which yields $q = \frac{1}{3}$.

This yields the mixed strategy NE where player 2 sets $p = \frac{2}{3}$ and player 1 sets $q = \frac{1}{3}$.

Problem 1.12.
Proceed similarly to above. Mixed Strategy NE: Player 2 plays $L$ with probability $\frac{3}{4}$ and $R$ with probability $\frac{1}{4}$. Player 1 plays $T$ with probability $\frac{2}{3}$ and $B$ with probability $\frac{1}{3}$.

Problem 2.6.
Consider second stage subgame for any given choice of $q_1 \geq 0$ in stage 1.
In this subgame, players 2 and 3 simultaneously choose $q_2, q_3$ to maximize their profits given by

$$\pi_2 = (a - q_1 - q_2 - q_3)q_2 - cq_2$$
$$\pi_3 = (a - q_1 - q_2 - q_3)q_3 - cq_3$$

This is just a standard Cournot quantity game with two identical firms. Taking the derivative of $\pi_2$ with respect to $q_2$ and $\pi_3$ with respect to $q_3$ and setting
them equal to zero yields the reaction functions:

\[
q_2 = \frac{a - c - q_1 - q_3}{2} \\
q_3 = \frac{a - c - q_1 - q_2}{2}
\]

Solving these two equations simultaneously we obtain the Nash equilibrium of the stage 2 subgame (given \(q_1\)):

\[
q_2^*(q_1) = \frac{a - c - q_1}{3} = q_3^*(q_1).
\]

Now, we look at the reduced form game in stage 1. This is a one-player game. Firm 1 simply chooses the \(q_1\) that maximizes his profit

\[
\pi_1 = (a - q_1 - q_2^*(q_1) - q_3^*(q_1))q_1 - cq_1
\]

\[
= (a - c - q_1 - 2(\frac{a - c - q_1}{3}))q_1
\]

\[
= \frac{(a - c)}{3} - \frac{1}{3}q_1q_1
\]

Taking derivative with respect to \(q_1\) and setting it equal to zero we obtain:

\[
\frac{a - c}{3} - \frac{2}{3}q_1 = 0
\]

which yields the optimal choice in the first stage:

\[
\overline{q}_1 = \frac{a - c}{2}.
\]

The corresponding NE choices in the next stage are:

\[
\overline{q}_2 = q_2^*(\overline{q}_1) = \frac{a - c}{6}
\]

\[
\overline{q}_3 = q_3^*(\overline{q}_1) = \frac{a - c}{6}.
\]

Subgame perfect equilibrium: \(q_1 = \overline{q}_1, q_2 = \overline{q}_2, q_3 = \overline{q}_3\).

Question 1.
This is a simultaneous move game with two players viz., farmers 1 and 2.
The strategy of each player \(i\) is the number of goats \(g_i\) she puts on the commons.
The payoff of player \(i, i = 1, 2\), is given by

\[
\pi_i(g_i, g_j) = (\overline{G} - (g_i + g_j))g_i - cg_i
\]

Taking derivative with respect to \(g_i\) and setting it equal to zero we obtain:

\[
g_i = \frac{\overline{G} - c - g_j}{2}, i = 1, 2, j \neq i.
\]
This yields two equations in two unknowns solving which obtain the Nash equilibrium:

\[ g_1^* = g_2^* = \frac{G - c}{3}. \]

Thus, the total number of goats on the commons:

\[ G^* = g_1^* + g_2^* = \frac{2(G - c)}{3}. \]

The joint profit:

\[ \pi = \pi_1 + \pi_2 = (G - G)G - cG \]

where \( G = g_1 + g_2 \). Taking the derivative with respect to \( G \) and setting it equal to zero we obtain the joint profit maximizing solution:

\[ \hat{G} = \frac{G - c}{2}. \]

It is easy to check that \( \hat{G} < G^* \).

2. For any \( p_1 > c \) but not exceeding the monopoly price, it is optimal for firm 2 in the second stage to undercut firm 1 and charge \( p_2 \) slightly below \( p_1 \) (thus taking the whole market and earning strictly positive profit). If \( p_1 > \) monopoly price, it is optimal for firm 2 to simply charge monopoly price and earn monopoly profit. If \( p_1 \leq c \), firm 2 can optimally charge \( p_2 = c \). Therefore, the reduced form profit of firm 1 is \( < 0 \) if \( p_1 < c \) and \( = 0 \) if \( p_1 \geq c \). Thus, \( p_1 = p_2 = c \) is a backward induction outcome.

But there many other backward induction outcomes - e.g., firm 1 charging any price \( p_1 \in (c, \text{monopoly price}) \) and firm 2 charging a price slightly below \( p_1 \), or firm 1 charging \( p_1 > \text{monopoly price} \) and firm 2 charging the monopoly price. These latter outcomes point to a second mover advantage and a first mover disadvantage quite unlike the Stackelberg game.

3. If \( X \) spends \( < $999,999 \) in the first stage, \( Y \) will spend \$1 more and win the contract leaving \( X \) with negative payoff. If \( X \) spends \$999,999 in the first stage, it is optimal for \( Y \) to spend \$0 and \( X \) wins contract with net payoff \$1. If \( X \) spends \$1m, then it is optimal for \( Y \) to spend \$0 and \( X \) wins contract with payoff \$0.

Backward induction solutions: \( X \) spends \$999,999.

There is a mild first mover advantage.