1. Consider the following normal form game with two players where players are allowed to play mixed strategies:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>C</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>4,2</td>
<td>3,3</td>
<td>0,1</td>
</tr>
<tr>
<td>M</td>
<td>3,5</td>
<td>4,4</td>
<td>0,0</td>
</tr>
<tr>
<td>B</td>
<td>1,0</td>
<td>0,0</td>
<td>-1,-1</td>
</tr>
</tbody>
</table>

Derive a Nash equilibrium of the game.

There is no Nash equilibrium in pure strategies.

After eliminating strictly dominated strategies B and R, we consider a mixed strategy Nash equilibrium where Player 1 randomizes between T and M with probabilities $q$ and $1 - q$, $0 < q < 1$, while player 2 randomizes between L and C with probabilities $p$ and $1 - p$, $0 < p < 1$.

The expected payoff for player 1, given such a mixed strategy played by 2, is $4p + 3(1 - p)$ when he plays T and $3p + 4(1 - p)$ when he plays M. In order for it to be optimal for player 1 to randomize between T and M, the expected payoffs need to be equalized which yields $p = \frac{1}{2}$.

Likewise, looking at the expected payoff of player 2 from playing L and C we obtain $q = \frac{1}{2}$.

Thus, there is a unique Nash equilibrium and it is mixed strategies where Player 1 randomizes between T and M with probability $\frac{1}{2}$ each and player 2 randomizes between L and C with probability $\frac{1}{2}$ each.

2. Two firms are bidding in an auction for the right to mine a mineral deposit. Firm 1 can make a profit of $2 million by mining the deposit. Firm 2 has an inferior mining technology which allows it to make a profit of only $1 million by mining the deposit. The bidding game goes as follows. First, firm 1 announces its bid $b_1$. This is observed by firm 2 which then announces its bid $b_2$. The mineral deposit is awarded to the highest bidder and if both bids are equal, the deposit is divided evenly between both bidders (each firm getting half of the profit it would make if it got the right over the entire deposit). Only the winning bidder pays up (pays an amount equal to his bid). The payoff for each bidder is his expected net profit (net of the bid if he wins the auction); a bidder gets zero if he does not win. Assume that bids are in whole dollars. Derive the backward induction solution to this two stage game.

Consider stage 2 of the game. If $0 \leq b_1 < 999,999$, bidder 2 will find it optimal to bid $1 + b_1$ and win the auction. If $b_1 = 999,999$, bidder 2 has two optimal actions: bid $1m$ and win, or bid something less than 999,999 and lose the auction for sure, both yield zero net profit ((if he equalizes bidder 1’s bid, he gets $500,000 but has to pay his bid 999,999). If $b_1 = 1m$, the unique optimal action for bidder 2 is to bid something less than 999,999 and
lose the auction for sure. If $b_1 > 1m$, it is optimal for bidder 2, to bid less than $b_1$ and lose the auction.

Two backward induction solutions.
1) Bidder 1 bids 999,999 and bidder 2 bids something less than 999,999 so that bidder 1 wins for sure.
2) Bidder 1 bids 1$m$ and bidder 2 bids less than 1million so that bidder 1 wins for sure; further, for this to be the solution, if bidder 1 had bid 999,999, bidder 2 would have bid 1$m$ in stage 2.

3. Consider the following extensive form game:

\[
\begin{array}{c|cc}
 & L & R \\
\hline
1 & 3 & 4 \\
2 & 4 & 2
\end{array}
\]

(i) Write this game as a normal form game and derive the Nash equilibria (confining attention to pure strategies).

Strategy Set of Player 1: \{L, R\}
Strategy Set of player 2: \{l, r\}

Payoffs can be depicted in matrix form:

\[
\begin{array}{ccc}
& l & r \\
L & 3 & 4 \\
R & 4 & 2
\end{array}
\]

There are two NE in pure strategies: (L, r), (R, l)

Note: (L, l) is not a NE even though it yields same payoff as (L, r); playing L is not a best response for player 1 to player 2 playing l.

(ii) Is there a credibility problem with one of the Nash equilibria? Explain.

There is a credibility problem with the NE (L, r). If we look at the extensive form of the game, if player 2 has to actually make a decision between l and r, he would never choose r. The NE strategies (L, r) are based on a threat by player 2 to play r if player 1 chooses R; in this NE, player 2 makes this "threat" expecting player 1 to play L in this equilibrium and thus expecting its bluff to not be called (in the sense that 2 wouldn’t really have to make a choice). The threat is not credible in the sense it would not be consistent with what would be optimal for player 2 at his decision node if he were ever to be required to move there.

(iii) What is the backward induction solution?

In the last stage of the game, player 2 moves l optimally.
In the reduced form stage 1 game, firm 1 can foresee that it will obtain 4 in stage 2 if it chooses R.
It is therefore, optimal for player 1 to choose R in stage 1.
Backward induction solution (R, l).