1. Consider a monopolist who faces the following market demand curve:

\[ q = 10 - p, \quad 0 \leq p \leq 10 \]
\[ = 0, \quad p > 10, \]

The monopolist’s production technology exhibits constant returns to scale; the constant marginal and average cost of production is $2.

(a) For any quantity \( q \), \( 0 \leq q \leq 10 \), derive an expression for the marginal revenue of the seller when he sells \( q \) units.

(Hint: First write down the total revenue of the firm in terms of \( q \)).

(b) Derive the optimal output for the monopolist. Use this to derive the monopoly price and monopoly profit.

(c) What is the Lerner index of market power in this market?

(d) What would be the socially optimal output in this market? Compare this to the monopoly output.

(e) Calculate the deadweight loss (loss of social surplus) due to monopoly power.

(f) Suppose the government imposed a tax on the firm; in particular, the firm is required to pay a tax = $2 for each unit it sells. Thus its effective cost of supplying the good is now $4 per unit. What is the new monopoly price and output? What happens to the deadweight loss? Illustrate your answer in a diagram.

(g) If instead of a tax, the government offered a subsidy equal to $1 per unit of output to the firm. How would the monopoly output and deadweight loss change? Illustrate your answer in a diagram.

(h) Suppose the government imposes a price ceiling \( \bar{p} \) on the firm. Is there a price ceiling that can lead to the socially optimal output? What are the problems associated with such a price ceiling?

2. Consider a market where firms have the following production cost:

\[ C(q) = 1 + q, \quad q > 0. \]

(i) What is the average and marginal cost of production?

(ii) Is this a natural monopoly?

Now, suppose that the market demand is given by:

\[ q = \frac{4}{p}, \quad p > 0. \]
(iii) What is the socially optimal level of output?
(iv) Suppose the government wants to impose a price ceiling that leads to the socially optimal level of output. What price ceiling should it set and what, if any, lump sum subsidy would it need to offer to get the firm to produce this output?
(v) Suppose the government wants to impose a price ceiling using average cost pricing. What price ceiling should it impose?
(vi) What are the pros and cons of the regulation scheme in part (v)?