OLIGOPOLY.

The Simplest Model of Price Competition in a Duopoly: The Bertrand Model.

The Symmetric Bertrand Model in a Homogenous Good Market.

Two identical firms: 1,2,. 

Identical product.

Constant Returns to Scale: Unit cost of production = c (for both firms).

Market demand curve: \( D(p) \) downward sloping, smooth.

Game: Firms set their prices \( p_1, p_2 \) simultaneously.

The quantity sold by each firm depends on both prices.
In particular, if \( p_1 < p_2 \), then firm 1 sells the entire market demand \( D(p_1) \) while firm 2 sells zero.

The reverse happens when \( p_1 > p_2 \).

If \( p_1 = p_2 = p \) (say), then each firm sells \( \frac{1}{2} D(p) \).

Nash equilibrium: A pair of prices \((\hat{p}_1, \hat{p}_2)\) such that neither firm can do better by unilaterally deviating and charging some other price.

There are several possibilities:

1. A pair of prices \((\hat{p}_1, \hat{p}_2)\) where \( \hat{p}_1 > \hat{p}_2 > c \).

   In this situation firm 1 earns zero profit. Firm 1 would be better off unilaterally deviating and charging a price \( p_1 \) just below \( \hat{p}_2 \). That way, it (firm 1) would make strictly positive profit. So this cannot be a NE.
2. A pair of prices \((\hat{p}_1, \hat{p}_2)\) where \(\hat{p}_2 > \hat{p}_1 > c\) : ruled out similarly.

3. A pair of prices \((\hat{p}_1, \hat{p}_2)\) where \(\hat{p}_1 = \hat{p}_2 = \hat{p}(say) > c\)

In this case firm 1 earns profit \(= \frac{1}{2}D(\hat{p})(\hat{p} - c)\)

If it unilaterally deviates a charge a price slightly below \(\hat{p}\), its profit will be approximately (slightly below) \(D(\hat{p})(\hat{p} - c)\), almost double of what it was getting. So, this cannot be a NE.

4. A pair of prices \((\hat{p}_1, \hat{p}_2)\) where \(\hat{p}_1 > \hat{p}_2 = c\).

In this situation both firms earn zero profit. Firm 2 would be better off unilaterally deviating and charging a price \(p_2\) just below \(\hat{p}_1\) but higher than \(c\). That way, it (firm 2) would make strictly positive profit. So this cannot be a NE.

5. A pair of prices \((\hat{p}_1, \hat{p}_2)\) where \(\hat{p}_2 > \hat{p}_1 = c\): ruled out similarly.
6. A pair of prices \((\hat{p}_1, \hat{p}_2)\) where \(\hat{p}_1 = \hat{p}_2 = c\).

In this case, both firms earn zero profit.

Further, no firm can deviate and do better i.e., strictly positive profit.

This is a NE.

It is the unique NE.

*If firms are identical, the unique NE is one where firms set their prices = MC and the industry output is at the socially optimal level. Both firms earn zero profit.*

Even with two firms - market power disappears.

If more than 2 firms - same outcome.

Bertrand paradox.
Assumptions of the Bertrand: rule out certain important elements of real markets.

1. Product Differentiation:

In Betrand model, firms sell identical product

⇒ firms can grab all buyers from rival by charging a slightly lower price (undercutting).

Real world: differentiated products

⇒ consumers may not switch to another firm simply because it charges a lower price (care about difference in product attributes)

⇒ incentive for a firm to undercut rival’s price is much less.

2. Dynamic competition:
Bertrand model is static (one shot game): prices chosen only once.

Real world rival firms have long term interaction - repeatedly choose prices over time.

Repeated games: better outcomes than in static games.

Possibility of future "retaliation" (through price war) can dissuade firms from undercutting now.

3. Capacity Constraints:

Bertrand model: firms can produce as much as they want at the same unit cost.

Real world: firms have limited production capacity.

Undercutting rival’s price to attract more buyers is not very useful if the firm cannot produce more output to meet the increased demand.
Consider a simple modification of the symmetric Bertrand duopoly model where each firm has a capacity constraint.

Let $k_1, k_2$ denote the capacity constraints of firms 1 and 2.

A firm cannot sell more than its capacity.

Firms produce output at constant unit cost up to their capacity.

For simplicity, set unit production cost $= 0$ for both firms.

Firms set prices simultaneously.

With capacity constraints, undercutting rival does not necessarily reduce rival’s sales to zero; there may be a positive residual demand left for the rival.

For example, suppose $p_1 > p_2$. 
Ideally, all consumers want to buy from firm 2.

If \( D(p_2) \leq k_2 \), then firm 2 sells the entire market demand \( D(p_2) \) and firm 1 sells zero.

If, however, \( D(p_2) > k_2 \), then firm 2 sells \( k_2 \) which means there are consumers who want to buy from firm 2 but are turned away. Some of them may be willing to buy from firm 1 at price \( p_1 > p_2 \). So, firm 1 can sell quantity \( D(p_1) - k_2 \) (or zero, if \( D(p_1) < k_2 \) which corresponds to a situation where none of the consumers turned away by firm 1 are willing to pay the higher price).

In other words, for and price \( p_1 > p_2 \), the residual market demand curve facing firm 1 is \( D(p_1) - k_2 \) i.e., a horizontal left-ward shift of the market demand curve by an amount of \( k_2 \).

Consider a situation where the total industry capacity \( k_1 + k_2 \) is "small" relative to market demand.
Let $\bar{p}$ be the price at which market demand is exactly equal to $k_1 + k_2$

$$D(\bar{p}) = k_1 + k_2$$

What are the NE prices?

If $k_1 + k_2$ is small enough relative to market demand, NE: $p_1 = p_2 = \bar{p}$.

Why?

Suppose firm 2 charges $\bar{p}$.

If firm 1 charges $p_1 = \bar{p}$, its profit (= revenue, here) is $\bar{p}k_1$.

If it charges $p_1 < \bar{p}$, it sells the same quantity = its capacity $k_1$(though consumers are willing to buy more). Selling same quantity at lower prices can never increase profit.
If it charges $p_1 > \bar{p}$, then we have situation where $p_1 > p_2$ and as discussed above, the residual market demand facing firm 1 is $D(p_1) - k_2$.

Check diagrammetically, that if $k_1 + k_2$ is "small" relative to market demand, then the marginal revenue curve corresponding to this residual market demand is strictly positive.

$MR > 0$ implies total revenue (= profit, here) increases if output is increased. Therefore, firm 1 is better off reducing its price. In other words, firm 1 cannot gain by increasing price above $\bar{p}$.

This proves that firm 1’s best response to $p_2 = \bar{p}$, is to charge $p_1 = \bar{p}$.

Vice-versa.

Thus, $p_1 = p_2 = \bar{p}$ is a NE.
Note that if \( k_1 + k_2 \) is small, \( \bar{p} \) is large and so firms have a lot of market power (price is considerably higher than \( MC = 0 \)). Firms 1 and 2 make profit equal to \( k_1 \bar{p} \) and \( k_2 \bar{p} \) respectively.

What if \( k_1 + k_2 \) is large relative to market demand?

If firms split the market by charging equal prices, their capacity is not fully utilized.

So, each firm undercuts rival to gain consumers.

Get back the Bertrand outcome.

Conclusion: More severe the capacity constraints, less competitive the market and higher the extent of market power. Industries with large capacity behave very competitively.

An Example of Price Competition with Capacity Constraints.
Market Demand:

\[ q = 100 - p \]

The inverse demand

\[ p = 100 - q \]

There are two firms. Each with capacity constraint = 20 (i.e., \( k_1 = k_2 = 20 \)).

For simplicity assume that both firms have zero unit cost of production.

Firms set prices simultaneously.

Claim: \( p_1 = p_2 = $60 \) is a Nash Equilibrium.
Note 60 is the price at which market demand is just sufficient to meet the industry output when both firms produce at full capacity.

Why is this a NE?

Suppose firm 2 charges \( p_2 = \$60 \).

If firm 1 charges \( p_1 = \$60 \), the two firms split the market equally and each firm sells 20 units leading to a profit of \$1200 for the firm.

If firm 1 charges \( p_1 < \$60 \), it is the lower priced firm, it faces demand of more than 40 units but can only sell 20 units. Its profit is \( 20p_1 < \$1200 \).

If firm 1 charges \( p_1 > \$60 \), it is the higher priced firm.

All consumers go to buy from firm 2 at price $60 & their demand is 40 units. But firm 2 can only sell 20 units.
In order to figure out how many units will be bought from firm 1, we look at the residual market demand after 20 units are sold by firm 2. This is given by

\[ q = (100 - p) - 20 = 80 - p \]

Firm 1 is like a monopolist facing this residual demand.

From our discussion of elasticity of demand recall that for a straight line demand curve of this kind, price elasticity of demand is greater than 1 at all points above the midpoint (and \( MR > 0 \)).

The midpoint of this (residual) demand curve (40,40). So definitely at any price above $60, demand is elastic and \( MR > 0 \).

This means firm can increase total revenue by reducing price.
Therefore, raising \( p_1 \) above $60 cannot be more profitable (remember, here total revenue = profit).

Conclusion: \( p_1 = $60 \) is a best response to \( p_2 = $60 \).

Similarly, the converse holds true.

Therefore, \( p_1 = p_2 = $60 \) is a Nash Equilibrium.

**Cournot Model of Quantity Competition.**

Previous analysis: If production capacity is limited, then firms will set prices at a level such that the market demand exactly equals the total production capacity.

In the story so far: capacity is exogenously fixed.

But what levels of capacity will competing firms choose?

Higher the capacity, more intense the price competition and smaller the price eventually charged in the market.
Need a model of endogenous capacity choice ⇒ Cournot oligopoly model.

In the Cournot model, firms choose their levels of output (or equivalently, their capacity to produce output) and it is assumed that the price at which they sell is one at which market demand equals industry output (or, industry capacity).

**Simple Symmetric Cournot Duopoly Model:**

Two identical firms, $i = 1, 2$.

Both firms produce at constant unit cost $c \geq 0$.

Market demand: $D(p)$

Inverse demand: $P(q)$

Note $P(q)$ gives us the demand price corresponding to total quantity $q$. 
Strategies of firms: output $q_1, q_2$. (interpret as capacity)

Industry output: $q_1 + q_2$

Then, the price in the "market" is $P(q_1 + q_2)$

Profit of firm 1 (its payoff): $\pi_1(q_1, q_2) = q_1 P(q_1 + q_2) - cq_1$

Profit of firm 2 (its payoff): $\pi_2(q_1, q_2) = q_2 P(q_1 + q_2) - cq_2$

Note that each firms’s profit depends on both its output as well as its rival’s output (through the inverse market demand function that determines the price).

Other things being equal, if one firm increases its output, it reduces its rival’s profit because it reduces the price at which output is sold.
While a firm’s profit maximization process takes into account the effect of its output on its own profit, it does not take into account the effect on rival’s profit. This basic externality eventually hurts them both.

Nash equilibrium (NE): A pair of output \((\tilde{q}_1, \tilde{q}_2)\) such that:

(i) given (belief about) firm 1’s output \(q_1 = \tilde{q}_1\), firm 2 maximizes its profit \(\pi_2\) by choosing \(q_2 = \tilde{q}_2\)

(ii) given (belief about) firm 2’s output \(q_2 = \tilde{q}_2\), firm 1 maximizes its profit \(\pi_1\) by choosing \(q_1 = \tilde{q}_1\)

In order to derive the NE, we first derive the optimal (or profit maximizing) action of each firm for each possible output of the rival firm. This gives us the "best-response curve" or the "reaction function" of each firm.

By definition, a NE \((\tilde{q}_1, \tilde{q}_2)\) is a point that lies on
the best-response curve for firm 1 because $\tilde{q}_1$ is firm 1’s best response to $\tilde{q}_2$

the best-response curve for firm 2 because $\tilde{q}_2$ is firm 2’s best response to $\tilde{q}_1$

i.e., its the intersection of the two best response curves or reaction functions.

Deriving the reaction function for firm 2:

Fix any output level $q_1 = \overline{q}_1$ for firm 1.

Then, the residual demand faced by firm 2 is given by a horizontal leftward shift of the market demand curve by an amount $\overline{q}_1$.

Firm 2 looks at the marginal revenue curve corresponding to this residual demand curve & equates it to the marginal cost $c$. 
This determines the best response output of firm 2.

If $\bar{q}_1 = 0$, then the residual demand faced by firm 2 is simply the market demand curve and so the point at which its MR equates MC is simply the monopoly output $q^m$.

So, the best response to zero output by rival, is to produce monopoly output.

For higher values of $\bar{q}_1$, the residual market demand is lower, the corresponding MR curve is even lower and the point at which MR equates MC for firm 2 is smaller.

At $\bar{q}_1 = q^S$, even if firm 2 produces zero, the market price is $c$ (because market demand is equal to marginal cost $c$ at $q^S$) and so firm’s best response is zero.

So, in a graph where we measure the outputs of firms 1 and 2 on the two axes, the reaction function of firm 2 is a downward sloping curve whose intercept on the
$q_2 - axis$ is the monopoly output $q^m$ and the intercept on the other axis is $q^S$.

Note: $q^m < q^S$

Similarly, we can derive the reaction function of firm 1.

The point of intersection: NE.

Observe graphically that the point of intersection $(\tilde{q}_1, \tilde{q}_2)$:

(i) lies above the straight line given by the equation $q_1 + q_2 = q^m$

(ii) lies below the straight line given by the equation $q_1 + q_2 = q^S$

Therefore, $q^m < \tilde{q}_1 + \tilde{q}_2 < q^S$
This means that at the NE of the Cournot game, industry output lies between monopoly output and socially optimal output.

The price in the market at the NE outputs: \( P(\tilde{q}_1 + \tilde{q}_2) = \tilde{p}(say) \).

Then,

\[
p^m < \tilde{p} < c
\]

Deriving NE analytically:

Consider the specific demand function:

\[
q = 40 - p
\]

Suppose \( c = 10 \).

Step 1: Derive the reaction function for firm 2.
Suppose $q_1 = \overline{q}_1$.

The residual demand faced by firm 2:

$$q_2 = (40 - p) - \overline{q}_1$$

which can be re-written as:

$$p = 40 - \overline{q}_1 - q_2$$

so that its total revenue corresponding to any output $q_2$:

$$TR_2 = (40 - \overline{q}_1 - q_2)q_2$$

$$= (40 - \overline{q}_1)q_2 - (q_2)^2$$

and using our rule for finding slope of a function (treat $\overline{q}_1$ as just a constant), we have that the marginal revenue for firm 2 (here $q_2$ is the variable) is given by:

$$MR_2 = (40 - \overline{q}_1) - 2q_2$$
Putting $MR_2 = MC$:

$$(40 - \bar{q}_1) - 2q_2 = 10$$

we have:

$$q_2 = 15 - \frac{\bar{q}_1}{2}$$

More generally, for any $q_1$ set by firm 1, the best response of firm 2 is

$$q_2 = 15 - \frac{q_1}{2}$$

This is the reaction function of firm 2.

Step 2: Derive the reaction function for firm 1.

Suppose $q_2 = \bar{q}_2$.

The residual demand faced by firm 1:

$$q_1 = (40 - p) - \bar{q}_2$$
which can be re-written as:

\[ p = 40 - q_2 - q_1 \]

so that its total revenue corresponding to any output \( q_1 \):

\[
TR_1 = (40 - q_2 - q_1)q_1 \\
= (40 - q_2)q_1 - (q_1)^2
\]

and using our rule for finding slope of a function (treat \( q_2 \) as just a constant), we have that the marginal revenue for firm 1 (here \( q_1 \) is the variable) is given by:

\[
MR_1 = (40 - q_2) - 2q_1
\]

Putting \( MR_1 = MC \):

\[
(40 - q_2) - 2q_1 = 10
\]

we have:

\[
q_1 = 15 - \frac{q_2}{2}
\]
More generally, for any $q_2$ set by firm 2, the best response of firm 1 is

$$q_1 = 15 - \frac{q_2}{2}$$

This is the reaction function of firm 1.

Step 3:

Find the point of intersection $(\tilde{q}_1, \tilde{q}_2)$ of the two reaction functions.

Such a point must lie on both reaction functions i.e.,

$$\tilde{q}_1 = 15 - \frac{\tilde{q}_2}{2}$$

$$\tilde{q}_2 = 15 - \frac{\tilde{q}_1}{2}$$
Solve these two equations simultaneously for two unknowns \((\tilde{q}_1, \tilde{q}_2)\).

Solution: \(\tilde{q}_1 = \tilde{q}_2 = 10\).

Industry output in equilibrium = \(\tilde{q}_1 + \tilde{q}_2 = 20\).

Equilibrium price: \(\tilde{p} = 40 - 20 = 20\)

Equilibrium profit of each firm = 100.

Comparison:

Monopoly price and output in this market: \(p^m = 25, q^m = 15\).

Socially optimal output: \(q^S = 30\).

**Asymmetric Cost Oligopoly.**
In the models of oligopoly we looked at so far, firms are identical.

What if firms differ in their efficiency or productivity?

Production costs may differ.

**Bertrand Model with Asymmetric Cost.**

2 firms: 1 and 2.

Both produce at constant unit cost.

Marginal cost of firm 1: $c_1$

Marginal cost of firm 2: $c_2$

Firms compete in prices - as in the standard model of Bertrand price competition (no capacity constraints).
We have seen that if \( c_1 = c_2 \), unique NE is that both firms set price equal to MC and earn zero profit.

What if \( c_1 < c_2 \)?

First, note that firm 2 can never make money in any NE.

To make money, \( p_2 > c_2 \) in which case firm 1 will find it optimal to undercut firm 2.

However, if firm 1 charges a price \( p_1 \) just below \( c_2 \), it will never be matched or undercut by firm 2 and it can make money.

This is actually the NE if \( c_1 \) and \( c_2 \) are not too far apart.

But if \( c_2 \) is extremely high relative to \( c_1 \), firm 1's monopoly price will be below \( c_2 \) and in that case firm 1 simply charges monopoly price.
In both cases, the relatively inefficient firm (firm 2) gets wiped out of the market.

This is a consequence of severe competition.

Note that if \( c_1 \) and \( c_2 \) are "close", then even though firm 2 gets zero market share and firm 1 has 100 % of the market - it cannot charge a monopoly price - threat of "potential competition" can discourage monopoly power.

**Cournot Model with Asymmetric Cost.**

Cournot model: oligopolistic competition is softer.

So, there is greater scope for a relatively inefficient firm to produce profitably in the market.

2 firms: 1 and 2.

Both produce at constant unit cost.
Marginal cost of firm 1: $c_1$

Marginal cost of firm 2: $c_2$

Firms compete in quantity of output (or capacity) - as in the standard Cournot model.

Consider the reaction function of any firm, say firm 1.

Given output $q_2 = \overline{q}_2$ of its rival, firm 1 looks at the residual demand curve and equates marginal revenue corresponding to that residual demand to its MC $c_1$.

Lower its own marginal cost $c_1$ is, higher the "best response" output of firm 1.

In other words, if $c_1$ decreases the entire reaction function or best response curve of firm 1 shifts up.

So, lets suppose that initially we have a symmetric situation where $c_1 = c_2$ so that the two best response curves
are mirror images of each other and at the NE, both firms produce identical output.

Now, if firm 1’s marginal cost $c_1$ falls slightly i.e. we move to a situation where $c_1 < c_2$, its reaction function shifts up.

The point of intersection of the two best response curves i.e., the NE moves to a point where firm 1 has a higher market share and not surprisingly, higher profit. But firm 2 still has a positive market share.

If $c_1$ falls drastically, so that its reaction shifts up in a big way, there may no longer be any positive intersection between the two reaction functions.

In that case, firm 1 produces its monopoly output and firm 2 produces zero.