Investment, Externalities
and Industry Dynamics

(Very preliminary and incomplete)

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ABSTRACT. We provide an alternative theoretical explanation for a number of empirical regularities relating to the dynamics of industry structure (product life cycle) and changes in size and age distribution of firms over time. We explain why entry may continue over a considerable period of time, why shake out of firms occur in mature industries and why exiting firms are likely to be younger and smaller in size than incumbents. Unlike the existing theoretical literature, this explanation is not based on uncertainty, structural non-stationarity or incomplete information. We consider an infinite horizon, complete information, deterministic competitive industry with continuum of firms and stationary market demand. Firms have perfect foresight, may enter or exit the industry at any point of time and active firms undertake investment which reduces their future cost of production. Investment by active firms also leads to the growth of an industry-wide capital that reduces production cost of all firms (externality). The marginal cost curves are upward sloping and firms incur a fixed cost of staying in the industry. While all entering firms earn zero intertemporal net profit, their instantaneous net profit is typically negative when they are young and strictly positive when they mature. Positive profits may persist in the long run. Equilibrium prices decline over time while the level of positive industry-wide externality increases with time. The equilibrium path makes firms indifferent between alternative entry and exit decisions and their investment levels after entry reflect their length of stay and the nature of industry environment (prices, externalities) over their period of stay in the industry. Heterogeneity emerges out of deliberate choice.
1 Introduction.

Economists have long recognized that the dynamics of industry structure as well as firm size & performance are closely related to changes in technology and productivity. The latter, in turn, are related to activities such as investment in learning-by-doing, cost reducing innovations and other forms of capital (including knowledge capital) that take place at the level of individual firms. While some of these activities clearly result in assets that are firm-specific (for example, organizational capital), others lead to creation of assets (such as knowledge) that spillover to other firms in the industry. The stocks of these various forms of capital and the intensity of productivity enhancing investments change over time, causing changes in the size or production "scale" of firms and thereby affecting prices & profitability in the industry. The latter, in turn, determine the incentives for entry and exit of firms and thus, the industry structure. In other words, the dynamics of industry structure, size & performance of firms and productivity changes in the industry are interlinked. In this paper, we analyze the dynamics of a competitive industry where firms may enter or exit the industry over time, carry out investment in capital that reduces firm specific production cost and, at the same times, generates positive industry-wide externalities.

It is now generally understood that industries experience very high turnover of firms and exhibit high degree of variance in size and growth rates across firms. Over the last few decades, empirical studies of technologically progressive manufacturing industries have established certain broad regularities pertaining to the manner in which industries evolve from birth through maturity that have collectively come to be known as the product life cycle. These regularities relate, among other things, to the pattern of entry, exit and growth of firms within industries as well as changes in the size & age distribution of firms. In the early phase of an industry, there is a lot of entry. In some industries, the number of entrants may rise over time or it may peak at the start of the industry and then decline over time. In either case, the number of entrants eventually becomes small and shake-out of firms begins as the industry matures. The number of active of firms grows initially, then reaches a peak, after which it declines steadily despite continued growth in industry output. Eventually, the industry stabilizes. The other set of empirical regularity has to do with age and size distribution of firms. Firms that enter earlier are more likely to grow faster, tend to be larger in size and have a greater chance of survival. On the average, firms that exit the industry are smaller and younger than the incumbents.

Since the early eighties, a wide range of theoretical models of stochastic evolution and selection in competitive industries have been developed in order

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1 For example, Dunne, Roberts and Samuelson (1989) study a sample of US manufacturing industries over a period of 5 years and report rates of entry ranging from 30.7% to 42.7% and an equally dramatic exit rate ranging from 30.8% to 39% across industries. See also, Davis and Haltiwanger (1992).
2 Gort and Klepper (1982) and Klepper and Grady (1990) examine the annual time path in the number of producers for 46 new products beginning with their commercial inception. See, also later studies by Utterback and Suarez (1993) and Klepper and Simons (1993).
3 For a nice summary, see Klepper (1996).
to explain the empirical regularities relating to industry dynamics. Thus, Jovanovic (1982) analyzes the dynamics of a competitive industry where firms are uncertain about their productivity and acquire noisy signals about their efficiency as they operate in the industry; incumbent firms afflicted by unfavorable signals conclude they are inefficient and exit the market to be replaced by new entrants - the efficient grow and survive, while the inefficient decline and fail (see also, Lippman and Rumelt, 1982). Pakes and Ericson (1998) discuss the implications of a more general version of this model and compare this with those of a stochastic model of their own, where firms actively undertake investment in order to influence the conditional distribution of future technology shocks affecting them.4 Klepper and Graddy (1990) discuss an evolutionary model where the number of potential entrants is limited, potential entrants differ in their initial cost and product qualities, receive new information over time which changes their cost and product quality in a stochastic fashion and no further updating of cost and quality occurs after entry.5 Jovanovic and Lach (1989) consider a model with learning by doing and stochastic diffusion of innovation where potential entrants can gain by learning from incumbent firms but all learning stops after entry; the model generates delayed entry and staggered exit.

In a fairly general model with exogenous firm level technology shocks and allowing for a wide class of firm-level actions (including investment in firm-specific cost reduction), Hopenhayn (1992a,b) shows the possibility of entry and exit as part of the limit behavior of a dynamic stochastic industry. In a similar model, Hopenhayn (1993) related the observed pattern of entry and exit over product life cycle to stochastic demand expansion and technology shocks. Jovanovic and MacDonald (1994) analyze a dynamic competitive industry where innovational opportunities fuel entry and failure to innovate, whose chances are exogenously specified, leads to exit.

The unifying feature of almost the entire existing literature is its reliance on some form of firm-level uncertainty (including uncertainty arising incomplete information & noisy signals) in creating and amplifying heterogeneity among firms. These shocks may affect either potential entrants or incumbents, or both. The process of market selection leads to exit of incumbent firms afflicted by unfavorable shocks (or signals) while entry occurs because of favorable updating of future profitability by potential entrants or simply because the prior belief about future profitability is significantly better than that of the firms that exit.6 To put

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4 Ericson and Pakes (1995) analyze a general model with stochastic entry and growth of firms that invest in order to improve future profitability & are affected by idiosyncratic shocks; they establish the ergodicity of a rational expectations Markov-perfect equilibrium process for the industry.

5 Klepper (1996) discusses a stochastic evolutionary model where firms differ randomly in innovative capabilities (relating to product innovation) and the value of innovation is proportional to output. The model allows for imitation as well as process innovation that only reduces current cost. The paper generates several observed features of the product life cycle - particularly, the sequencing of product and process innovations.

6 In a somewhat related exercise, Lambson (1991) analyzes a dynamic competitive model where firms make investment that entails sunk cost and whose profitability is affected by exogenous shocks over time; the equilibrium path exhibits hysteresis and high turnover of plants (see also Dixit, 1989).
it more bluntly, if all uncertainty is taken out of these models, then the industry equilibrium paths hardly generate any kind of interesting dynamics and quite often, reduce to the outcome of a static model.

This, then leads to the following question: is it the case that the patterns of changes in industry structure as observed say, over the product life cycle, arise only through uncertainty and that there are no other fundamental forces affecting industry dynamics? This paper is an attempt to provide an alternative explanation which is not based on any form of uncertainty, non-stationary demand structure or incomplete information.

We consider a classical, infinite horizon, deterministic, complete information model of a competitive industry with a continuum of firms. All firms are ex ante identical, perfectly rational, forward looking and have perfect foresight. There are no strategic factors affecting entry or causing exit. Further, unlike some of the existing literature, there is no dearth of potential entrants at any point of time. We pose the following questions: Can the dynamic path of an industry be consistent with the empirical regularities mentioned earlier?. Would some firms enter later than others? Would some firms exit earlier than others? Would later entrants tend to exit earlier? Would exiting firms be small relative to incumbents that stay on? Would firms become heterogeneous over time through deliberate choice even though they are initially identical? Can profits persist in the long run? This paper is an answer to all of these questions. And its an affirmative answer.

We build on earlier work by Petrakis, Rasmusen and Roy (1997) and Petrakis and Roy (1999) that analyze similar deterministic models of dynamic competitive industry. In these papers, firms that enter the industry invest in firm-specific cost reduction through accumulation of capital or experience. The industry equilibrium path is socially optimal and generates shake-out of firms. However, these papers are unable to explain some of the interesting empirical regularities such as the fact that entry continues over a considerable period of time i.e., some firms enter later than others, later entrants tend to be smaller and have a lower survival rate (i.e., exit earlier) so that exiting firms are typically smaller and younger and so on. One reason behind this is the fact that these models rule out spillover from investment by existing firms. Also, the models assume finite time horizon which rules out analysis of long run behavior of industries.

We consider a model which is similar to Petrakis and Roy (1999) - firms invest in cost reduction. However, in our model, the cost of production is also affected by the stock of an industry-wide capital that grows over time according to the total investment effort by all firms in the industry. The introduction of this externality complicates the analysis considerably - particularly, because the properties of the equilibrium path can no longer be related to a social planner’s optimization problem.

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Stokey (1986) analyzes a model of dynamic oligopoly with industry-wide externality where the unit cost of production depends on cumulative past output of the industry. There is no investment in firm-specific cost reduction nor is there any possibility of entry or exit in her model.
In equilibrium, all entering firms earn zero intertemporal net profit. However, their instantaneous profit, net of investment cost, is typically negative when they are young and strictly positive when they mature. Equilibrium prices decline over time, while the level of positive industry-wide externality increases with time - the former discourages delayed entry while the latter encourages it. The equilibrium path makes firms indifferent between alternative entry and exit decisions. Their investment levels after entry reflect their duration of stay and the nature of industry environment (prices, externalities) over their period of stay. Heterogeneity emerges out of deliberate choice. We show the possibility of delayed entry and shake-out of firms on the equilibrium path and relate them to the technology and demand conditions. Entry may occur even while incumbents earn negative profits. Exit may occur even while incumbents grow in size and earn positive profits. Exiting firms are smaller and younger than incumbents. Under certain conditions, entry and exit decline to zero in the limit and the industry stabilizes. Firms that are active in the long run may earn strictly positive profit in the limit (explains persistence of profits in the long run).

2 The Model.

Consider an infinite horizon perfectly competitive industry producing a homogeneous good. Time is discrete and indexed by $t = 0, 1, 2, \ldots \infty$. There is a continuum of price-taking firms that are free to enter and exit the industry in any period. The space of all firms is $\mathbb{R}_+$ endowed with the Lebesque measure. All firms are ex ante identical.

Besides producing output, entering firms may also invest in accumulation of firm specific capital (knowledge, organizational capital). The process of accumulating and retaining this form of capital is intricately tied to the firm being active in the industry. Firms lose their firm specific capital when they exit the industry. Thus, a firm that exits the industry is indistinguishable from a new firm that has not yet entered the industry. Also, firms cannot accumulate capital before entering the industry. As the measure of potential firms is infinite, we assume without loss of generality, that each firm enters at most once. Apart from firm specific capital, there is also an industry-wide knowledge capital that accumulates through industry-wide learning. The accumulation of this kind of capital is related to the total investment activity in the industry in each time period.

In every time period $t$, the current cost of production for any active firm $i$ depends on its current level of output (denoted by $q_t(i)$), firm specific capital (denoted by $z_t(i)$) as well the current stock of industry-wide capital, denoted by $K_t$. In particular, we let $C(q, z, K)$ denote the cost function. Besides production cost, a firm also incurs the cost of adding to its stock of firm specific capital. Let $x_t(i)$ denote investment made by firm $i$ in period $t$ and let $\phi(x)$ denote the investment cost function. Let $\tau(i)$ and $T(i)$ denote the periods of entry and exit for firm $i$. We adopt the convention that a firm enters at the beginning (prior to production) of period $\tau(i)$ and exits at the end of (after production in) period
\( T(i) \). Also, \( \tau(i) = \infty \) if firm \( i \) does not enter and \( T(i) = \infty \), if firm \( i \) does not exit the industry in finite time. Let \( S_t \) denote the set of firms that are active in period \( t \) i.e.,

\[ i \in S_t \iff \tau(i) \leq t \leq T(i) \]

In equilibrium, \( S_t \) is a measurable set. The accumulation of firm specific capital for any firm \( i \) is defined by:

\[ z_t(i) = \begin{cases} 0, & t \leq \tau(i) \text{ and } t > T(i) \\ z_{t-1}(i) + x_{t-1}(i), & \tau(i) < t \leq T(i). \end{cases} \tag{1} \]

The accumulation of industry-wide capital is defined by

\[ K_0 = 0, \]
\[ K_t = K_{t-1} + \Gamma \left( \int_{S_t} x_{t-1}(i) di \right) \tag{2} \]

where \( \Gamma(.) \) is the net industry wide learning in any period that depends on total investment activity in the industry in that period. Let \( p_t \) denote the market price in period \( t \). Each firm takes the market prices \( \{p_t\}_{t=0}^{\infty} \) as given and also anticipates (correctly, in equilibrium) the sequence of industry-wide capital \( \{\tilde{K}_t\} \). For any firm \( i \), its choice of entry period \( \tau(i) \), exit period \( T(i) \) and its profile of output and investment \( \{q_t(i), x_t(i)\}_{t=\tau(i)}^{T(i)} \) during its stay in the industry defines its payoff viz., the intertemporal net profit:

\[ \sum_{t=\tau(i)}^{T(i)} \delta^t [p_t q_t(i) - C(q_t(i), z_t(i), \tilde{K}_t) - \phi(x_t(i))]. \tag{3} \]

All firms make these choices so as to maximize their intertemporal net profit. These choices define a sequence of realized industry output \( \{Q_t\}_{t=0}^{\infty} \):

\[ Q_t = \int_{S_t} q_t(i) di \]

and a sequence of realized industry-wide capital stock \( \{K_t\}_{t=0}^{\infty} \) in the manner described in (2).

The following assumptions are imposed on the production cost function \( C(q,z,K) \):

(A.1) \( C(q,z,K) \) is continuous on \( \mathbb{R}_+^3 \) and twice differentiable in \( (q,z) \);

(A.2) For any \( K \geq 0 \), \( C(q,z,K) \) is strictly increasing in \( q \) and strictly convex in \( (q,z); C_q > 0 \).

(A.3) \( C(q,z,K) \) and \( C_q(q,z,K) \) are both non-increasing in \( z \) and \( K \);

(A.4) \( f = \lim_{z \to -\infty, K \to -\infty} C(0, z, K) > 0 \).

(A.5) For each \( q \geq 0 \), \( m(q) = \lim_{z \to -\infty, K \to -\infty} C_q(q, z, K) > 0 \); \( m(q) \) is strictly increasing in \( q \) and \( m(q) \to \infty \) as \( q \to \infty \).
(A.1) is a standard smoothness assumption. (A.2) implies that in each period, the marginal cost of production is strictly positive and strictly increasing in output. The upward sloping marginal cost curve reflects the fact that its stock of knowledge (firm specific as well as industry-wide) is fixed in each period. Strict convexity of \( C \) in \( (q,z) \) is used to ensure uniqueness of solution to a firm’s dynamic profit maximization problem (for given entry and exit periods). (A.3) indicates that an increase in firm specific capital and/or industry-wide capital leads to a decline (weakly) in the firm’s cost of production i.e., the average cost curve shifts downwards as \( z \) or \( \bar{K} \) increase. Further, the marginal cost curve is weakly decreasing in \( z \) and \( \bar{K} \). Since the marginal cost curve is the firm’s supply curve, our assumptions imply that a firm’s supply is weakly expanding in capital stock. Our assumptions allow for situations where capital accumulation reduces only the fixed cost of production as well as situations where they reduce only the marginal cost curve. (A.4) ensures that for each period a firm stays in the industry, it incurs a fixed cost. The fixed cost may be reduced by capital accumulation but is bounded away from zero. Incurring such a cost is necessary in order for the firm to hold on to its stock of firm specific knowledge and organizational capital. It is avoided only by exiting the industry. The combination of positive fixed cost and upward sloping marginal cost curve implies that the average cost curve is U-shaped in each period. (A.5) imposes a bound on the extent of dynamic scale economies by requiring that the lowest possible marginal cost (that one can possibly reach through capital accumulation) is strictly positive and becomes infinitely large as output expands to infinity. This ensures that output and investment are adequately bounded for a competitive equilibrium to exist.

The investment cost function \( \phi(x) \) satisfies the following restrictions:

(A.6) \( \phi \) is strictly increasing and continuously differentiable on \( \mathbb{R}_+ \), \( \phi(0) = 0 \), \( \phi'(0) > 0 \).

(A.7) \( \phi \) is convex on \( \mathbb{R}_+ \).

Note that we allow for the possibility that \( \phi \) is linear.

The market demand in each period depends only on the price in that period and is identical over time. In particular, the market demand curve is given by a function \( D(p) \) and the inverse demand function is denoted by \( P(Q) \). We assume that:

(A.8) \( D(p) \) is continuously differentiable, strictly decreasing and \( D'(p) < 0 \) for all \( p > 0 \). Further, for any \( Q > 0 \), \( \int_0^Q P(y) \, dy < \infty \), where \( P = D^{-1} \).

The net industry-wide learning function \( \Gamma \) satisfies the following restriction:

(A.9) \( \Gamma(0) = 0 \), \( \Gamma \) is strictly increasing and continuous on \( \mathbb{R}_+ \).

For any sequence of strictly positive prices \( p = \{p_t\}_{t=0}^\infty \) and sequence of anticipated industry-wide capital \( \bar{K} = \{\bar{K}_t\}_{t=0}^\infty \), consider the intertemporal profit maximization problem of firm that enters in period \( \tau \geq 0 \) and exits in period
\[ T \geq \tau \ (T \text{ may be } +\infty) : \]
\[
\max_{q_t, x_t, t=\tau \ldots T} \sum_{t=\tau}^{T} \delta^t \{ p_t q_t - C(q_t, z_t, \hat{K}_t) - \phi(x_t) \}
\]
\[
q_t \geq 0, \quad z_{\tau} = 0, \quad z_{t+1} = z_t + x_t, \quad t = \tau, ... T - 1.
\] (4)

It is easy to check that under the above assumptions:

**Lemma 1** There exists a unique solution to the maximization problem (4).

Thus, all firms that enter and exit in the same periods choose the same output and investment vectors over their periods of stay in the industry. The equilibrium path can therefore be described by measures of firms that choose

Let \( q(p, \bar{K}, \tau, T) \) and \( x(p, \bar{K}, \tau, T) \) respectively denote the unique output and investment vectors (or, sequences) \( \{ q_t \}_{t=\tau}^{T}, \{ x_t \}_{t=\tau}^{T} \) that solve (4) and let \( \Pi(p, \bar{K}, \tau, T) \) denote the maximum intertemporal profit generated by this solution. The relevant first order necessary conditions for optimality are given by:

\[
p_t = C(q_{t,T}, z_{t,T}, \hat{K}_t), \quad \text{if} \quad q_{t,T} > 0
\] (5)

\[
\leq C(q_{t,T}, z_{t,T}, \hat{K}_t), \quad \text{if} \quad q_{t,T} = 0
\] (6)

\[
\phi'(x_{t,T}) + \sum_{j=t+1}^{T} \delta^{j-t} C_z(q_{j,T}, z_{j,T}, \hat{K}_j) = 0, \quad \text{if} \quad x_{t,T} > 0
\] (7)

\[
\geq 0, \quad \text{if} \quad x_{t,T} = 0
\] (8)

Let \( N_+ \) be the set of non-negative natural numbers and let the set \( \nu \) be defined by

\[
\nu = \{ (\tau, T) : \tau \in N_+, T \in N_+ \cup \{ +\infty \}, \tau \leq T \}
\]
denote the set of non-negative natural numbers augmented by the point \{ +\infty \}. We are now ready to define the notion of industry equilibrium.

**Definition 2** An Industry Equilibrium is given by

(a) a price sequence \( p = \{ p_t \}_{t=0}^{\infty} \),

(b) a sequence of industry wide knowledge \( \bar{K} = \{ \hat{K}_t \}_{t=0}^{\infty} \),

(c) a sequence \( \{ n_{\tau,T} \}_{\tau,T \in \nu} \) where \( n_{\tau,T} \) indicates the measure of firms that enter in period \( \tau \) and exit in period \( T \)

(d) action profile \( \{ q_{\tau,T}^t, x_{\tau,T}^t \}_{t=\tau}^{T} \) for each \( (\tau, T) \in \nu \) for which \( n_{\tau,T} > 0 \)

such that:

(i) \( K_t = K_{t-1} + \Gamma(\sum_{(\tau,T) \in \nu, \nu \leq t \leq T} n_{\tau,T} x_{\tau,T}^t), \quad K_0 = 0 \)
(ii) Firms choose output and investment at the intertemporal profit maximizing level: If \( n_{\tau T} > 0 \), then \( \{q_{\tau T}^t\}_{t=\tau}^T = q(p, K, \tau, T), \{x_{\tau T}^t\}_{t=\tau}^T = x(p, K, \tau, T) \).

(iii) Every active firm earns zero intertemporal profit: If \( n_{\tau T} > 0 \), then \( \Pi(p, K, \tau, T) = 0 \).

(iv) No incentive to change entry, exit decision: For every \((\tau, T) \in \nu\), \( \Pi(p, K, \tau, T) \leq 0 \).

(v) Market clears every period:

\[
D(p_t) = \sum_{\{(\tau, T) \in \nu : \tau \leq t \leq T\}} n_{\tau T} q_{\tau T}^t
\]

Note that (iii) and (iv) implies that if \( n_{\tau T} > 0 \), then for all \( j, \tau \leq j \leq T \)

\[
\sum_{t=j}^T \delta^t \{p_t q_{\tau T}^t - C(q_{\tau T}^t, z_{\tau T}^t, K_t) - \phi(x_{\tau T}^t)\} \geq 0
\]  

(9)

We now introduce some more notations. Let the minimum average cost of production for any level of firm specific and industry-wide capital \((z, K)\) be denoted by \(A(z, K)\) i.e.,

\[
A(z, K) = \min_{q \geq 0} \frac{C(q, z, K)}{q}
\]  

(10)

Using assumptions (A.2) – (A.4), it is easy to check that:

**Lemma 3** \(A(z, K)\) is non-increasing in \(z\) and \(K\) and \(A(z, K) \geq m(0) > 0, \forall z, K \geq 0\).

Thus, the lowest possible average cost is strictly positive and we denote this by \(\underline{p}\):

\[
\underline{p} = \lim_{z \to -\infty, K \to -\infty} A(z, K) > 0
\]  

(11)

We denote by \(\overline{p}\), the minimum average cost for a new entrant firm in period 0:

\[
\overline{p} = A(0, 0)
\]  

(12)

It is easy to check that: \(0 < \underline{p} < \overline{p} < +\infty\). The following assumption ensures that market demand is strictly positive in every period on the equilibrium path:

(A.10) \(D(\overline{p}) > 0\).

Define \(\overline{Q}, \underline{Q}, \overline{\pi}, \underline{\pi}\) by

\[
\overline{Q} = D(p), \underline{Q} = D(\overline{p})
\]  

(13)
We are now ready to state our result on the existence of equilibrium:

**Proposition 4** An industry equilibrium exists (under assumptions (A.1)–(A.10)).

**Proof.** See Appendix. ■

In the rest of this paper, we will implicitly assume that (A.1)–(A.10) hold.

### 3 Basic Properties of Equilibrium Path.

In this section, we state some useful properties of an industry equilibrium. We begin with an important monotonicity property of equilibrium price paths:

**Proposition 5** In any industry equilibrium, the price sequence \( \{ p_t \} \) is non-increasing over time i.e., \( p_{t+1} \leq p_t \), \( \forall \ t \geq 0 \).

**Proof.** Suppose to the contrary that \( p_{t+1} > p_t \) for some \( t \geq 0 \). If \( n_{\tau t} > 0 \) for some \( \tau \leq t \) (i.e., exit occurs in period \( t \)) then

\[
p_t q^t_{\tau t} - C(q^t_{\tau t}, z^t_{\tau t}, K_t) \geq 0
\]

(for otherwise, condition (iii) of the definition of industry equilibrium implies \( \tau < t \) and \( \Pi(p, K, \tau, t - 1) > 0 \), a contradiction). This, in turn, implies that

\[
p_{t+1} q^t_{\tau t} - C(q^t_{\tau t}, z^t_{\tau t}, K_{t+1}) > 0
\]

(since \( K_{t+1} \geq K_t, p_{t+1} > p_t \)), which means any firm exiting in period \( t \) can earn strictly positive profit by staying on for one more period with no additional investment i.e., \( \Pi(p, K, \tau, t + 1) > 0 \) which violates the definition of industry equilibrium. Therefore, it must be the case that \( n_{\tau t} = 0 \) for all \( \tau \leq t \). Thus, the measure of active firms in the industry in period \( t + 1 \) is at least as large as in period \( t \). Further, since \( p_{t+1} > p_t, K_{t+1} \geq K_t \) and for all \( (\tau, T) \) such that \( \tau \leq t \leq T, z^t_{\tau t} \leq z^{t+1}_{\tau t} \), the first order conditions of profit maximization (5) and (6) imply that

\[
q^t_{\tau t} > q^{t+1}_{\tau t} \text{ if } q^{t+1}_{\tau t} > 0
\]

\[
q^{t+1}_{\tau t} = q^t_{\tau t} \text{ if } q^{t+1}_{\tau t} = 0
\]
so that industry output $Q_t < Q_{t+1}$. Condition (iv) of the definition of industry equilibrium then implies that $p_t > p_{t+1}$, a contradiction.

The next result establishes bounds on prices, output and investment on the equilibrium path. Recall, the definition of critical values $p, \bar{p}, Q, \bar{Q}, \bar{q}, \bar{x}, \bar{n}$ in the previous section.

**Proposition 6** In any industry equilibrium, $p_t \in [p, \bar{p}]$ and $Q_t \in [Q, \bar{Q}]$, $\forall t \geq 0$.

The output produced and the investment made by any firm in any period are uniformly bounded above by $q$ and $x$, respectively i.e., $q^T_T \leq \bar{q}, x^T_T \leq \bar{x}, \forall (\tau, T) \in \nu$. In any period $t \geq 0$, the measure of $S_t$, the set of active firms in the industry, given by $(\sum_{(\tau, T) \in \nu, \tau \leq t \leq T} n_{\tau T})$ is bounded above by $\pi$. Finally, $K_{t+1} - K_t \leq \bar{n}x$.

**Proof.** If $p_t > \bar{p}$ for some $t \geq 0$, then $\Pi(p, K, t, t)$ > 0 which contradicts condition (iv) of the definition of equilibrium. If $p_t < p$ for some $t \geq 0$, then $p_t < p$ for all $t' \geq t$ (using Proposition 5) i.e., all active firms earn strictly negative profit every period after $t$ and therefore (using (9)), $\tau \leq t \leq T$ implies $n_{\tau T} = 0$. Thus, $Q_t = 0$. Assumption (A.10) implies quantity demanded is strictly positive in period $t$ which yields a contradiction. $p_t \in [p, \bar{p}]$ and $Q_t = D(p_t)$ implies $Q_t \in [Q, \bar{Q}]$. Since $p_t \leq \bar{p}$, from (5) and assumption (A.3) we have $q^T_T \leq \bar{q}$. Also, for any active firm, the reduction in production cost in any period obtained by investment is bounded above by $(\bar{p} - p)\bar{q}$ and therefore, from (16), we have $x^T_T \leq \bar{x}$. Finally, from (15), observe that if the measure of active firms in the industry in any period $t$ is strictly greater than $\pi$, then the total production cost in the industry in that period strictly exceeds $\frac{\pi}{1+\delta}$, the highest possible discounted sum of total revenue in the industry from that period onwards. This implies (9) is violated for a positive measure of active firms in period $t$, a contradiction.

### 4 Entry Dynamics.

Our model of industry dynamics provides an explanation of the fact that entry of firms is dispersed over time instead of being concentrated at the birth of the industry. Our analysis will show that even when there is a sufficiently large mass of potential entrants at the birth of the industry and all potential entrants are ex ante identical, free entry at the birth of the industry does not eliminate the market incentives for later entry. Indeed, since prices are declining on the equilibrium path, it would seem imprudent for a firm to enter later. The reason that entry may occur in later periods in our framework is because of the existence of an industry-wide capital that confers a positive externality on individual firms - new entrants as well as incumbents - and the magnitude of this externality changes over time. In particular, the increase in industry-wide knowledge over time offsets the effect of declining prices. As a result, even though entry in
period zero continues to the point where firms entering in period zero break even, it leaves open the scope for gainful entry at a later time.

If firms never invest in firm specific capital, the industry equilibrium path is simply a repeated version of the static equilibrium in which case no entry (or, for that matter, exit) occurs after period zero. We begin by introducing an assumption that ensures that firms make strictly positive investment in firm specific capital in their period of entry:

(A.11) \( \phi'(0) + \delta C_z(q, 0, K) < 0, \forall q \in [0, \bar{q}], K \geq 0. \)

**Lemma 7** Under (A.11), \( T > \tau \) and \( n_{-T} > 0 \) implies \( x_{\tau-T} > 0. \)

**Proof.** Follows from (8). ■

The next result states a very useful fact about delayed entry viz, that it cannot be preceded by exit of firms. In other words, exit cannot be immediately followed by entry.

**Lemma 8** Assume (A.11). If strictly positive measure of firms enter in period \( \tau > 0 \), then the measure of firms that exit the industry in period \( (\tau - 1) \) is zero.

**Proof.** Suppose that \( n_{-T} > 0 \), for some \( \tau > 0 \), \( T \geq \tau \) and contrary to the lemma, \( n_{\tau-1} > 0 \) for some \( t \leq \tau - 1 \). Consider a firm that enters in period \( t \) and exits in period \( T \) with the following action profile:

\[
q_j = \begin{cases} 
q^{j}_{\tau-1}, & j = t, \ldots, \tau - 1 \\
q^{j}_{\tau}, & j = \tau, \ldots, T 
\end{cases}
\]

\[
x_j = \begin{cases} 
x^{j}_{\tau-1}, & j = t, \ldots, \tau - 2 \\
x^{j}_{\tau}, & j = \tau, \ldots, T 
\end{cases}
\]

\[
x_{\tau-1} = 0 = x^{\tau-1}_{\tau-1},
\]

if \( t < \tau - 1 \) (i.e., \( z^{\tau-1}_{\tau-1} > 0 \))

\[
x_{\tau-1} = \epsilon,
\]

if \( t = \tau - 1 \) (i.e., \( z^{\tau-1}_{\tau-1} = 0 \))

where \( \epsilon > 0 \) is small enough. Using the facts that \( \Pi(p, K, \tau - 1) = \Pi(p, K, \tau, T) = 0, z_{\tau} = \sum_{j=t}^{\tau-1} x_j > 0 \) and assumption (A.11), it is easy to check that the firm following the above strategy can make strictly positive i.e., \( \Pi(p, K, t, T) > 0 \), a contradiction. ■

To emphasize the crucial role played by industry-wide externality (and more generally, spillover of knowledge) in generating dispersal of entry over time, we begin by showing that in the absence of such an externality, all entry occurs in period zero. A similar result is contained in Petrakis and Roy (1999) for the finite horizon case.
Proposition 9 Assume (A.11). Suppose that the cost of production is independent of industry-wide capital i.e., $C(q, z, K)$ is independent of $K$. Then, all entry occurs in period zero i.e., $n_{t-T} = 0, \forall T > 0$.

Proof. Suppose to the contrary that $\exists t > 0$ such that $n_{t-T} > 0$ for some $T \geq t$. From Lemma 8, we have that the measure of firms exiting in period $t - 1$ is zero. If $p_{t-1} = p_t$, then since no exit occurs at the end of period $t - 1$, every active firm in period $t - 1$ produces at least as much in period $t$ and further, a strictly positive measure of firms enter in period $t$ so that $Q_{t-1} < Q_t$, which violates market clearing. Thus, $p_{t-1} > p_t$. Since production cost is independent of $K$, it is easy to verify that following an action profile

$$q_j = q_1^{j+1}, x_j = x_2^{j+1}, z_j = z_3^{j+1}, j = t - 1, ... T - 1,$$

the net intertemporal profit of a firm entering in period $t - 1$ and exiting in period $T - 1$ (where $T - 1 = \infty$) evaluated in terms of present value at period $t - 1$, is given by

$$\sum_{j=t-1}^{T-1} \delta^{j-(t-1)}[p_j q_j - C(q_j, z_j) - \phi(x_j)]$$

as $p_{t-1} > p_t, p_j \geq p_{j+1}, j \geq t$,

$$\sum_{j=t-1}^{T-1} \delta^{j-(t-1)}[p_{j+1} q_{j+1} - C(q_{j+1}, z_{j+1})] - \phi(x_{j+1})]$$

$$\sum_{j=1}^{T} \delta^{j-1}[p_j q_{j+1} - C(q_{j+1}, z_{j+1}) - \phi(x_{j+1})] = 0$$

so that $\Pi(p, K, t-1, T-1) > 0$, which contradicts the definition of equilibrium.

Thus, the existence of industry-wide knowledge spillover is important for generating temporal dispersal of entry. In industries, where such spillover or externalities are small, one should expect that, in the absence of external shocks (such as technological change), entry ought to be concentrated around the birth of an industry.

Next, we argue that what matters for delayed or dispersed entry of firms over time is the manner in which production cost is affected by firm specific and industry-wide capital. It is intuitive that when capital formation largely affects variable costs and, in particular, leads to sharp decline in the marginal cost curves of firms i.e., large outward shifts in their supply function as well as their efficient scale of production, then entry is not very likely. Such forces are more likely to generate shakeout of firms. On the other hand, if capital formation leads to significant decline in the fixed cost and relatively small shifts in the marginal cost (so that the efficient scale actually declines), then we are more likely to see entry of firms over time. We formalize this idea in terms of a sufficient condition for entry to occur infinitely often in the industry.
As we have seen above, a necessary condition for delayed entry is that production cost be sensitive to industry-wide knowledge formation. Therefore, we assume:

(A.12) For any \( q \in [0, \overline{q}] \), \( z \geq 0 \), \( C(q, z, K) \) is strictly decreasing in \( K \).

Let \( \xi \) be the lower bound on the absolute slope of the demand curve:

\[
\xi = \inf_{p \in [\underline{p}, \overline{p}]} |D'(p)|
\]

Proposition 10 Assume (A.11) and (A.12). Suppose that for any \( q \in [0, \overline{q}] \), \( z \geq 0 \), \( K \geq 0 \) and \( h \in (\frac{p}{\underline{p}}, 1) \), the following holds

\[
C_q(q + \frac{\xi h}{\overline{p}}, z + \overline{p}, K + \overline{q}) - C_q(q, z, K) > -h
\]

Then, entry occurs infinitely often in the industry i.e., for each \( t \geq 0 \), there exists \( \tau \geq t \) such that \( n_{\tau \rightarrow T} > 0 \) for some \( T \geq \tau \).

Proof. We know that strictly positive measure of entry occurs in period 0. It is sufficient to show that if strictly positive measure of entry occurs in period \( t' \), then it strictly positive measure of entry must occur in some period \( t > t' \). Suppose to the contrary that there exists \( t' \geq 0 \) such that, \( n_{t' \rightarrow T} > 0 \) for some \( T \geq t' \) and no entry occurs in every period \( t \geq t' \). We first claim that in that case, \( p_t = p_{t+1}, \forall t \geq t' \). To establish this claim, let \( t \) be the first time period \( \geq t' \) for which \( p_t > p_{t+1} \). Let \( h \) be defined by

\[
p_t = p_{t+1} + h
\]

Then, \( h \in (0, \overline{p} - \underline{p}) \). Further,

\[
Q_{t+1} - Q_t = D(p_t - h) - D(p_t) \geq \xi h
\]

so that

\[
Q_{t+1} \geq Q_t + \xi h.
\]

As no entry occurs in period \((t + 1)\), there must be a strictly positive measure of active firms in period \( t \) whose output increases by at least as much as \( \frac{\xi h}{\overline{p}} \) between periods \( t \) and \( t + 1 \), i.e., for some \( \tau, T, \tau \leq t, T \geq t + 1 \):

\[
qu_{\tau \rightarrow T}^{t+1} \geq qu_{\tau \rightarrow T}^t + \frac{\xi h}{\overline{p}}
\]

(18)

This implies \( qu_{\tau \rightarrow T}^{t+1} > 0 \). Using the first order conditions (5) and (6)

\[
p_{t+1} = C_q(q_{\tau \rightarrow T}^{t+1}, z_{\tau \rightarrow T}^{t+1}, K_{t+1})
\]

\[
p_t \leq C_q(q_{\tau \rightarrow T}^t, z_{\tau \rightarrow T}^t, K_t)
\]
and $p_{t+1} = p_t - h$ we have
\[
C_q(q_{t+1}^T, z_{t+1}^T, K_{t+1}) \leq C_q(q_t^T, z_t^T, K_t) - h
\]

and using (18) we have
\[
C_q(q_{t+1}^T, z_{t+1}^T, K_{t+1}) \leq C_q(q_t^T, z_t^T, K_t) - h
\]

and since
\[
C_q(q_{t+1}^T, z_{t+1}^T, K_{t+1}) \geq C_q(q_t^T, z_t^T, K_t) - h
\]

we have
\[
C_q(q_{t+1}^T, z_{t+1}^T, K_{t+1}) \leq C_q(q_t^T, z_t^T, K_t) - h
\]

which contradicts (17). This establishes our claim that $p_t = p_{t+1}, \forall t \geq t'$. Next, observe that if $n_{t'} > 0$, then $T > t'$. For if $T = t'$, then $p_{t'} = A(0, K_{t'}) = p_t, \forall t \geq t'$. Using (A.11) and $K_{t'+1} \geq K_t$, it is easy to check that $\Pi(p, K, t') = 0$, a contradiction. Thus $T > t'$. This implies (use Lemma 7) that $x_{T+1}^T > 0$ and therefore $K_{t'} < K_{t'+1}$. Using assumption (A.12), it is easy to check that $\Pi(p, K, T+1) = 0$, (where $T + 1 = \infty$, if $T = \infty$). This contradicts the definition of industry equilibrium. The proof is complete. ■

One situation in which (17) always holds is when the accumulation of firm specific and industry-wide capital only affect the fixed cost of production and the marginal cost curve is unchanged over time i.e., $C(q, z, K)$ is of the form $F(z, K) + G(q)$ where $F$ is the fixed cost of production and $G$ is the variable cost.

**Corollary 11** Assume (A.11) and (A.12) and that the marginal cost of production (and therefore, the variable cost) is independent of firm specific capital as well as industry-wide capital Then, (strictly positive measures of) firms enter the industry infinitely often.

**Proof.** Since $C_q$ is strictly increasing in $q$ and independent of $(z, K)$, the left hand side of (17) $> 0 > -h$. ■

5 Shake-out.

In this section, we discuss the possibility of shake-out of firms in the industry and the economics of why some firms exit earlier than others. We show that even though there is no uncertainty or incomplete information and even though all firms are identical *ex ante*, the industry equilibrium path can be characterized
by some firms choosing to stay forever while others choose different finite periods of exit.

The core argument here is that if the marginal cost curve declines (firms’ supply curve expands) sharply with increase in knowledge, then in the absence of shake-out, the market clearing prices would have to fall drastically (the intensity of this fall also depends on elasticity of market demand). But if the prices fall too sharply, firms cannot recover their investment through future profits (prices faced by mature firms need to be more than their minimum average cost). Equilibrating forces in the industry therefore ensure exit of firms over time.

Another way to see the phenomenon is that the market is "restricted efficient" in the sense that taking equilibrium path of industry wide knowledge \( \{K_t\} \) as given, the output, investment, entry and exit decisions of firms on the equilibrium path maximizes the discounted sum of social surplus. As the marginal cost curve is upward sloping and steep initially, a net surplus maximizing social planner wants to divide production among many firms when the industry is young (despite the fixed cost) in order to reduce industry production cost but may want only some of them to grow "big" and produce in later periods (with relatively flat marginal cost curves).

We begin by stating a result that illustrates the fact that for exit to occur it is necessary that capital formation reduces marginal cost (i.e., variable cost).

**Lemma 12** Assume (A.11) and that the marginal cost of production (and therefore, the variable cost) is independent of firm specific capital as well as industry-wide capital. Then, the measure of firms that exit the industry in finite time is zero.

**Proof.** Let \( t \geq 0 \) be the first time period in which a strictly positive measure of firms exit. Using Lemma 8, the measure of firms entering in period \( t + 1 \) must be zero. Since the marginal cost \( C_q \) is independent of \( z, K \), all firms have identical supply functions in all periods. As \( p_{t+1} \leq p_t \) and exit occurs in period \( t \), the total industry output must be strictly smaller in period \( t + 1 \) than in period \( t \). However, \( D(p_{t+1}) \geq D(p_t) \). Thus, the market cannot clear, a contradiction. ■

Our next result shows that exit of firms must coincide with positive investment activity among incumbent firms.

**Lemma 13** If \( n_{\tau T} > 0 \) for \( \tau \leq T < \infty \), then there exists \( (\tau', T') \in \nu \), \( \tau' \leq T \leq T' \) such that \( n_{\tau' T'} > 0 \) and \( x_{\tau' T'} > 0 \). Thus \( K_T < K_{T+1} \).

**Proof.** Suppose not. From Lemma 8, we have that no entry occurs in period \( T + 1 \). Thus, the measure of active firms declines between periods \( T \) and \( T + 1 \) and the supply curve of individual firms remains unchanged (as there is no investment). As \( p_T \geq p_{T+1} \), the total quantity supplied \( Q_{T+1} < Q_T \). This violates market clearing. ■

Next, we outline a sufficient condition for exit of firms in finite time. Recall that for any \( z, K \), the minimum average cost of production is denoted by
$A(z, K)$ and that $\overline{p} = A(0,0)$. Let $q^m(z, K)$ be the output at which average cost of production is minimized when knowledge levels are $(z, K)$

$$C_q(q^m(z, K), z, K) = A(z, K)$$

**Proposition 14** Assume (A.11). Suppose that for any $(z, K) \geq 0$

$$\frac{D(\overline{p})}{q^m(0, 0)} > \frac{D(A(z, K))}{q^m(z, K)}$$

then there exists a positive measure of firms who exit in finite time.

**Proof.** Let $n_t$ denote the total measure of active firms in period $t$. Since $p_0 \leq \overline{p} = A(0,0)$, the output of all firms in period 0 is bounded above by $q^m(0, 0)$. Thus,

$$n_0 \geq \frac{D(A(0,0))}{q^m(0, 0)}$$

Suppose contrary to the proposition, the measure of firms exiting in finite time is zero. Then $n_t \geq n_0, \forall t \geq 0$. Since $n_0 > 0$, $\exists \delta_{0,0} > 0$, for firms to break even there must exist finite $t' > 0$ such that $p_{t'} > A(\delta_{0,0}, K_{t'})$. Fix such $t' > 0$. Since $p_{t'} = C_q(\delta_{0,0}^{t'}, \delta_{0,0}^{t'}, K_{t'})$, it is easy to check that $\delta_{0,0}^{t'} > q^m(\delta_{0,0}^{0,0}, K_{t'})$ and therefore

$$n_{t'} \leq \frac{D(p_{t'})}{\delta_{0,0}^{t'}} < \frac{D(A(\delta_{0,0}^{t'}, K_{t'}))}{q^m(\delta_{0,0}^{t'}, K_{t'})}$$

and using (19), we have $n_{t'} < n_0$, a contradiction. \(\blacksquare\)

6 **Firms that exit are smaller and younger.**

In this section, we outline some results on the age and size of exiting firms in relation to that of incumbent firms in the industry. In particular, our model generates predictions that match the broad empirical regularities on product life cycles that suggest exiting firms are on the average, younger and smaller than incumbents. We show that firms that enter later always hold smaller stock of firm specific capital compared to earlier entrants and therefore, have higher cost of production and their supply curve is typically to the left of the supply curves of earlier entrants. Thus, the size of later entrants is smaller than that of earlier entrants. Further, it can never be the case that it is optimal for an earlier entrant to exit even while a later entrant stays on in the industry (as the latter holds a smaller stock of firm specific capital).

In order to establish all of these results, we need to strengthen the convexity requirement on the investment cost function:

(A.13) $\phi$ is strictly convex on $\mathbb{R}_+$. 

We now establish a basic monotonicity property of the optimal stock of firm specific capital next period as a function of the stock of firm specific capital in the current period.
Lemma 15 Assume (A.14). Let \((\hat{z}_t, \hat{z}_{t+1})\) and \((\tilde{z}_t, \tilde{z}_{t+1})\) be the levels of firm specific knowledge for two firms that are active in the market in both periods t and t + 1. Then, \(\hat{z}_t < \tilde{z}_t \Rightarrow \hat{z}_{t+1} < \tilde{z}_{t+1}\).

Proof. Suppose to the contrary that \(\hat{z}_{t+1} \geq \tilde{z}_{t+1}\). Then
\[
\hat{z}_{t+1} \geq \tilde{z}_{t+1} \geq \tilde{z}_t > \hat{z}_t
\]

For a firm that is active in the market at the beginning of period \(t + 1\) with firm-specific knowledge stock equal to \(z\), let \(V_{t+1}(z)\) denote the (maximum) present value of intertemporal profits from period \(t + 1\) onwards (net of investment made in such periods). Let \(\pi_t(z_t)\) denote the maximum profit in period \(t\) (gross of investment cost incurred in period \(t\)) for a firm whose current knowledge stock is \(z_t\). Then,
\[
\hat{z}_{t+1} \in \arg\max \{\pi_t(\hat{z}_t) - \phi(z - \hat{z}_t) + \delta V_{t+1}(z) : z \geq \hat{z}_t\}
\]
\[
\tilde{z}_{t+1} \in \arg\max \{\pi_t(\tilde{z}_t) - \phi(z - \tilde{z}_t) + \delta V_{t+1}(z) : z \geq \tilde{z}_t\}
\]

Therefore:
\[
\pi_t(\tilde{z}_t) - \phi(\tilde{z}_{t+1} - \tilde{z}_t) + \delta V_{t+1}(\tilde{z}_{t+1}) \geq \pi_t(\hat{z}_t) - \phi(\hat{z}_{t+1} - \hat{z}_t) + \delta V_{t+1}(\hat{z}_{t+1})
\]
\[
\pi_t(\hat{z}_t) - \phi(\hat{z}_{t+1} - \hat{z}_t) + \delta V_{t+1}(\hat{z}_{t+1}) \geq \pi_t(\tilde{z}_t) - \phi(\tilde{z}_{t+1} - \tilde{z}_t) + \delta V_{t+1}(\tilde{z}_{t+1})
\]

so that
\[
-\phi(\hat{z}_{t+1} - \hat{z}_t) + \delta V_{t+1}(\hat{z}_{t+1}) \geq -\phi(\tilde{z}_{t+1} - \tilde{z}_t) + \delta V_{t+1}(\tilde{z}_{t+1})
\]
\[
-\phi(\tilde{z}_{t+1} - \tilde{z}_t) + \delta V_{t+1}(\tilde{z}_{t+1}) \geq -\phi(\hat{z}_{t+1} - \hat{z}_t) + \delta V_{t+1}(\hat{z}_{t+1})
\]

First, consider the case where \(\hat{z}_{t+1} > \tilde{z}_{t+1}\). From the above inequalities, we have
\[
\phi(\hat{z}_{t+1} - \hat{z}_t) - \phi(\tilde{z}_{t+1} - \tilde{z}_t) \leq \delta [V_{t+1}(\hat{z}_{t+1}) - V_{t+1}(\tilde{z}_{t+1})] \leq \phi(\hat{z}_{t+1} - \hat{z}_t) - \phi(\tilde{z}_{t+1} - \tilde{z}_t)
\]

which violates strict convexity of \(\phi\) as \(\hat{z}_t > \tilde{z}_t\). Next, consider the case where \(\hat{z}_{t+1} = \tilde{z}_{t+1}\). Let \(i\) denote the firm whose knowledge stock in period \(t\) is \(\tilde{z}_t\) and let \(j\) denote the firm whose knowledge stock in period \(t\) is \(\hat{z}_t\). As \(V_{t+1}(\tilde{z}_{t+1}) = V_{t+1}(\hat{z}_{t+1})\), the maximum discounted sum of profits net of investment cost between periods \(t + 1\) and \(T(i)\) for firm \(i\) must be exactly equal to the maximum discounted sum of profits net of investment cost between periods \(t + 1\) and \(T(j)\) for firm \(j\). This would mean that from period \(t + 1\) onwards, the output and investment path of firm \(j\) (and firm \(j\)'s exit period) is also optimal for firm \(i\). The first order condition for firm \(j\) with respect to its investment in period \(t\) implies (note \(\tilde{z}_{t+1} - \tilde{z}_t > 0\))
\[
\phi'(\hat{z}_{t+1} - \hat{z}_t) = -\sum_{k=t+1}^{T(j)} \delta^{k-t} C_z(q_k(j), z_k(j), K_k)
\]
and since \( \bar{z}_{t+1} - \bar{z}_t < \bar{z}_{t+1} - \bar{z}_t \)

\[
\phi'(\bar{z}_{t+1} - \bar{z}_t) < - \sum_{k=t+1}^{T(j)} q_{k} C_z(q_k(j), z_k(j), K_k)
\]

which implies that if firm \( i \) invests an amount slightly higher than \( \bar{z}_{t+1} - \bar{z}_t \) in period \( t \) and thereafter replicates the action profile of firm \( j \), its net intertemporal profit will be higher than its initial optimal path, a contradiction.  

We are now ready to show that later entrants must hold smaller stock of firm specific capital and produce less of output than earlier entrants.

**Lemma 16** Assume (A.11) and (A.13). Consider \((\tau, T), (\tau', T') \in \nu, \) where \( n_{\tau,T} > 0, n_{\tau'T'} > 0. \) Then, \( \tau < \tau' \leq T \implies z^T_{\tau,T} > z^T_{\tau',T'}, q^T_{\tau,T} \geq q^T_{\tau',T'}, \) t = \( \tau' \), ..., \( \min\{T, T'\} \).

**Proof.** Since \( T > \tau \), using Lemma 7, \( z^T_{\tau,T} > 0 = z^T_{\tau',T'} \). The result now follows from Lemma 15 by induction (that \( q^T_{\tau,T} \geq q^T_{\tau',T'} \) follows from (5)).

The fact that later entrants hold smaller stock of firm specific capital implies that their incentive to exit is higher than that of earlier entrants. However, this result requires that the production cost be strictly decreasing in firm specific capital:

(A.14) For any \( q \in [0, q^*], K \geq 0, C(q, z, K) \) is strictly decreasing in \( z \) on \( \mathbb{R}_+ \).

**Proposition 17** Assume (A.11), (A.13) and (A.14). If \( n_{\tau,T} > 0 \) for some \( \tau \leq T < \infty, \) then for \( j = \tau + 1, \ldots, T, \) firms entering in period \( j \) must exit no later than period \( T \) i.e., \( n_{jT} = 0, \forall t > T. \) Thus, in any period, exiting firms are never older than incumbent firms that do not exit.

**Proof.** Suppose to the contrary that there exists \( j, \tau + 1 \leq j \leq T \) such that \( n_{j'T'} > 0 \) for some \( T' > T. \) Using Lemma 16, we have \( z^T_{\tau,T} > z^T_{\tau',T'}. \) Using (9) and (A.14), a firm that enters in period \( \tau \) and exits in period \( T \) can earn strictly positive intertemporal profit by staying in the industry till period \( T' \) by choosing actions \( q_t = q^T_{j'T'}, x_t = x^T_{j'T'}, t = T, \ldots, T'. \)

It follows directly from Lemma 16 and Proposition 17 that:

**Proposition 18** Assume (A.11), (A.13) and (A.14). If \( n_{\tau,T} > 0 \) for some \( \tau \leq T < \infty, \) then the output produced by any firm in period \( T \) is at least as large as \( q^T_{\tau,T} \) i.e., exiting firms are never larger in size than any incumbent firm that does not exit. Further, the firm specific capital of any firm in period \( T \) is at least as large as \( z^T_{\tau,T} \) i.e., exiting firms are never more productive than any incumbent firm that does not exit.

Using the fact that exiting firms earn non-negative profit in their period of exit and Proposition 18, it follows that:
Corollary 19 Assume (A.11), (A.13) and (A.14). If \( n_{+T} > 0 \) for some \( \tau \leq T < \infty \), then for all \((\tau', T')\) such that \( \tau' \leq T \leq T' \) and \( n_{+T'} > 0 \), \( p_T \geq \bar{A}(z_T^{T'}, K_T) \). Further, if \( \tau' < \tau, T' \geq T \) then, \( p_T \geq \bar{A}(z_T^{T'}, K_T) > \bar{A}(z_T^{T'}, K_T) \).

The above results indicate that, loosely speaking, the process of competitive shakeout in the industry is likely to involve exit of smaller, less profitable firms while most incumbents earn higher profit (gross of investment cost), are larger in size, are engaged in investment activity that leads to aggressive expansion of their size and output in the future and the market price is typically lower after exit. These are also features that might be associated with anti-competitive and predatory behavior by large incumbents in an industry and might set off antitrust warning bells. Our analysis indicates that the same features may be present in the dynamic path of a competitive industry where no firm has any market power.

7 Long Run Properties: Convergence of Industry Structure.

In this section, we show that in the long run, entry and exit of firms decline to zero and the industry "stabilizes". This is consistent with empirical observations on long run industry dynamics.

As the equilibrium price sequence \( \{p_t\} \) is non-decreasing and bounded below by \( \underline{p} > 0 \), it converges. Let \( p^* > 0 \) be defined by

\[
\lim_{t \to \infty} p_t = p^*
\]

Thus, the equilibrium industry output in non-decreasing and

\[
\lim_{t \to \infty} Q_t = Q^* = D(p^*)
\]

The sequence of industry-wide capital \( \{K_t\} \) is also non-decreasing and we can define \( K^* > 0 \) by

\[
\lim_{t \to \infty} K_t = K^*
\]

Note that \( K^* \) may be \( +\infty \). In order to establish our result on convergence, we require an additional assumption that says that the slope of the marginal cost curve is uniformly bounded away from zero.

(A.15) There exists \( \theta > 0 \) such that

\[
C_{qq}(q, z, K) \geq \theta, \forall q \in [0, \bar{q}], z, K \geq 0
\]

Proposition 20 Assume (A.11), (A.13), (A.14) and (A.15). The total measure of firms that enter the industry later than any finite date \( t \), converges to zero as \( t \to \infty \). Thus, the measure of all firms that (ever) enter the industry is finite.
Proof. First, consider the set of entrants that do not exit in finite time. It is easy to see that
\[ \sum_{\tau = 0}^{\infty} n_{\tau \infty} \leq \overline{\pi} \]  
for otherwise there exists \( \hat{t} \geq 0 \) such that \( \sum_{\tau=0}^{\hat{t}} n_{\tau \infty} > \overline{\pi} \) so that the measure of active firms in the industry in period \( \hat{t} + 1 \) exceeds \( \overline{\pi} \) which violates Proposition 6. It follows from (20) that
\[ \sum_{\tau = \hat{t}}^{\infty} n_{\tau \infty} \longrightarrow 0 \text{ as } t \longrightarrow \infty. \]  

Next, consider the set of all entrants including ones that exit in finite time. If there exists finite time period \( t \), after which the measure of entrants that exit in finite time is zero, then the proposition obviously holds. So, suppose that there is a subsequence of time periods with strictly positive measure of entry of firms that exit in finite time. Recall that that \( q^m(z, K) \) is the output that minimizes average cost for a firm with firm specific capital \( z \) and when industry-wide capital is \( K \). Using assumptions (A.4) and (A.5), there exists \( \tilde{q} > 0 \) such that
\[ q^m(z, K) \geq \tilde{q}, \forall z, K \geq 0 \]  
Since every exiting firm must earn non-negative profit in its period of exit, its output in that period is at least as large as \( \tilde{q} \). Choose any small \( \epsilon > 0 \). There exists \( T(\epsilon) \), such that for all \( t \geq T(\epsilon), p_t \in [p^*, p^* + \epsilon] \), the industry output \( Q_t \in [D(p^* + \epsilon), D(p^*)] \) and
\[ Q_k - Q_t \leq [D(p^*) - D(p^* + \epsilon)], \forall k \geq t \]  
Define the sequence time period \( \{\tilde{t}_n, \tilde{t}_n^r\}_{n=1}^{\infty} \). Let \( \tilde{t}_1 = \min\{\tau > T(\epsilon) : n_{\tau T} > 0 \) for some \( T \geq \tau \} \) and \( \tilde{t}_1 = \min\{T \geq \tilde{t}_1 : n_{\tau T} > 0 \) for some \( \tau \leq T \}. \) Given, \( \tilde{t}_{n-1}, \tilde{t}_{n-1} \) we can define \( \tilde{t}_n = \min\{\tau > \tilde{t}_{n-1} : n_{\tau T} > 0 \) for some \( T \geq \tau \} \) and \( \tilde{t}_n = \min\{T \geq \tilde{t}_n : n_{\tau T} > 0 \) for some \( \tau \leq T \}. \). As entry of firms that exit in finite time occurs infinitely often, the sequence \( \{\tilde{t}_n, \tilde{t}_n^r\}_{n=1}^{\infty} \) is well defined. Observe that
\[ \tilde{t}_n \leq \tilde{t}_n < \tilde{t}_{n+1}, \forall n \geq 1 \]  
Using Proposition 17, note that if \( \tau_n = \max\{t \leq \tilde{t}_n : \text{entry occurs in period } t\} \) then \( \tau_n \geq \tilde{t}_n \) and a strictly positive measure of firms entering in period \( \tau_n \) must exit in period \( \tilde{t}_n \). Using Lemma 7, no exit occurs in period \( \tilde{t}_n - 1 \) and no entry occurs in period \( \tilde{t}_n + 1 \). Finally,
\[ Q_{T(\epsilon)} \leq Q_{\tilde{t}_n} \leq Q_{\tilde{t}_n^r} \leq Q^*, p_{\tilde{t}_n} \geq p_{\tilde{t}_n^r} \geq p_{\tilde{t}_{n+1}} \geq p^*, \forall n \geq 1 \]  
Let \( m_t \) denote the total measure of firms entering in period \( t \). The set of active firms in the industry in period \( \tilde{t}_n \) can be separated into two subsets: (i) \( \tilde{S}_n \) of firms that were active in the industry in period \( \tilde{t}_n - 1 \) and continue to be active
in period $\bar{t}_n$ and (ii) $\bar{S}_n$ of firms that enter in some period $\tau$, $\bar{t}_n \leq \tau \leq \bar{t}_n$ (these firms exit in some period $T \geq \bar{t}_n$ including the possibility that $T = \infty$). As argued above, all firms that exit in period $\bar{t}_n$ (includes a strictly positive measure of firms that enter in period $\tau_n$) produce at least $\bar{q}$ in that period. Further, using Proposition 17, all firms that enter in periods $t \in [\bar{t}_n, \bar{t}_n]$ but do not exit in period $\tilde{t}_n$ must have entered in some period $\leq \tau$, hold at least as large a stock of firm specific capital in period $\tilde{t}_n$ as firms entering in period $\tau$ and exiting in period $\tilde{t}_n$. Since the marginal cost curve is non-increasing in firm specific capital, it follows that all firms that enter periods $t \in [\bar{t}_n, \tilde{t}_n]$ but do not exit in period $\tilde{t}_n$ must also produce at least $\bar{q}$ in period $\tilde{t}_n$. Thus, every firm in the set $\bar{S}_n$ produce at least $\bar{q}$ in period $\tilde{t}_n$ and the measure firms in the set $\bar{S}_n$ is exactly equal to $\sum_{j=\tilde{t}_n}^{\bar{t}_n} m_j$. Thus, the industry output in period $\tilde{t}_n$

$$Q_{\tilde{t}_n} \geq \bar{q}[\sum_{j=\tilde{t}_n}^{\bar{t}_n} m_j] + \int_{\bar{S}_n} q_k(i) di$$

$$= \bar{q}[\sum_{j=\tilde{t}_n}^{\bar{t}_n} m_j] + \int_{\bar{S}_n} C_q^{-1}(p_{\tilde{t}_n}, z_{\tilde{t}_n}(i), K_{\tilde{t}_n}) di$$

$$\geq \bar{q}[\sum_{j=\tilde{t}_n}^{\bar{t}_n} m_j] + \int_{\bar{S}_n} C_q^{-1}(p_{\tilde{t}_n}, z_{\til\tau_{n-1}}(i), K_{\til\tau_{n-1}}) di$$

(since $z_{\tau_{n-1}}(i) \leq z_{\til\tau_{n}}(i), K_{\til\tau_{n}} \leq K_{\til\tau_{n}}$)

$$= \bar{q}[\sum_{j=\til\tau_{n}}^{\bar{t}_n} m_j] + \int_{\bar{S}_n} C_q^{-1}(p_{\til\tau_{n-1}}, z_{\til\tau_{n-1}}(i), K_{\til\tau_{n-1}}) di$$

$$- \int_{\bar{S}_n} \{C_q^{-1}(p_{\til\tau_{n-1}}, z_{\til\tau_{n-1}}(i), K_{\til\tau_{n-1}}) - C_q^{-1}(p_{\til\tau_{n}}, z_{\til\tau_{n-1}}(i), K_{\til\tau_{n}})\} di$$

$$\geq \bar{q}[\sum_{j=\til\tau_{n}}^{\bar{t}_n} m_j] + \int_{\bar{S}_n} C_q^{-1}(p_{\til\tau_{n-1}}, z_{\til\tau_{n-1}}(i), K_{\til\tau_{n-1}}) di$$

$$- \frac{\pi}{\theta}(p_{\til\tau_{n-1}} - p_{\til\tau_{n}}) \quad (\text{mean value theorem & assumption (A.16)})$$

$$= \bar{q}[\sum_{j=\til\tau_{n}}^{\bar{t}_n} m_j] + Q_{\til\tau_{n-1}} - \frac{\pi}{\theta}(p_{\til\tau_{n-1}} - p_{\til\tau_{n}})$$

so that

$$[\sum_{j=\til\tau_{n}}^{\bar{t}_n} m_j] \leq [Q_{\til\tau_{n-1}} - Q_{\til\tau_{n-1}} - \frac{\pi}{\theta}(p_{\til\tau_{n-1}} - p_{\til\tau_{n}})]$$
This implies that the total measure of all firms entering after period $T(\epsilon)$

$$\sum_{\tau=T(\epsilon)}^{\infty} m_j = \sum_{n=1}^{\infty} \sum_{j=t_n}^{\infty} m_j \leq \sum_{n=1}^{\infty} \left[ (Q_{t_n} - Q_{t_n-1}) + \frac{\pi}{\theta} (p_{t_n-1} - p_{t_n}) \right] \leq (Q^* - Q_{T(\epsilon)}) + \frac{\pi}{\theta} (p^* - pr(\epsilon)) \leq \left\{ D(p^*) - D(p^* + \epsilon) \right\} + \frac{\pi}{\theta} \epsilon$$

The proposition follows.

The next proposition characterizes the intensity of shake-out in the long run:

**Proposition 21** Assume (A.11), (A.13), (A.14) and (A.15). The measure of firms that exit the industry later than any period $t \geq 0$ converges to zero as $t \rightarrow \infty$. The total measure of all firms that ever exit the industry is finite.

**Proof.** Suppose not. Then the total measure of all firms that exit the industry over all time periods must be infinite. This is inconsistent with Proposition 20.

From Propositions 20 and 21, it follows immediately that the measure of active firms converges and thus, the industry stabilizes in the long run.

**Corollary 22** As $t \rightarrow \infty$, the measure of active firms in the market in period $t$ converges to some $n^* \in [n, \bar{n}]$. Entry and exit converges to zero in the long run.

### 8 Persistence of Profits.

In this section, we illustrate the possibility of long run persistence of positive profits in the industry. Our analysis thus provides a theoretical underpinning for a wide range of empirical observations on industrial profit rates being strictly positive over long horizons. It is worth emphasizing that in our model, there is no dearth of potential entrants or heterogeneity among them; the persistence of profits in the long run is simply an equilibrium property of competitive markets that generate fair returns for firms’ investment made in the past and spread these returns over time in order to secure low prices for consumers. Further, the persistence of strictly positive profit even while the measure of entry becomes negligible in the long run, may appear to be indicative of anti-competitive barriers to entry. Our analysis indicates that this may well be a feature of competitive market with zero market power where the market rewards firms for past investment.
The examples we develop are really a class of situations where capital formation (both firm specific and industry wide) affects only the fixed cost of production and not the variable cost. The marginal cost curve is, therefore, identical across firms and over time. We have discussed results for this class of industries in Proposition 9 and Lemma 12. Note that there exists a subsequence of time periods \( \{t'\} \), such that \( K_{t'} < K_{t'+1} \) and \( p_{t'} > p_{t'+1} \).

We assume that the cost function is of the form:

\[
C(q, z, K) = f(z, K) + g(q)
\]

where \( f(z, K) \) represents the fixed cost (of being active in the industry) while \( g(q) \) represents the variable cost. We assume that (A.1) – (A.11) hold. We also assume that \( f_z \) is non-decreasing in \( K \) and that \( \phi(x) = x \).

Under these assumptions, an active firm invests in only one period viz., its period of entry, and this investment level is also its long run firm specific capital i.e., \( n_{\tau T} > 0 \) implies \( x_{\tau T} = 0 \), \( t = \tau + 1, \ldots, T \). To see this, observe that (A.11) implies \( x_{\tau T} > 0 \). Suppose that \( t = \tau + k \) is the first period after period after \( \tau \) such that \( x_{\tau T} > 0 \). Then (7) implies

\[
1 + \sum_{j=\tau+1}^{T} \delta^{j-\tau} f_z (z_{\tau T}^j, K_j) = 0
\]

\[
1 + \sum_{j=\tau+k+1}^{T} \delta^{j-(\tau+k)} f_z (z_{\tau T}^j, K_j) = 0
\]

These imply that

\[
0 = 1 + \sum_{j=\tau+1}^{\tau+k} \delta^{j-\tau} f_z (x_{\tau T}^j, K_j) - \delta^k
\]

\[
\leq 1 - \delta^k + \sum_{j=\tau+1}^{\tau+k} \delta^{j-\tau} f_z (x_{\tau T}^j, K_{\tau+k}), \text{since } f_z \text{ is non-decreasing } K
\]

so that

\[
f_z (x_{\tau T}^\tau, K_{\tau+k}) \geq 1 - \frac{1}{\delta}
\]

On the other hand,

\[
-1 = \sum_{j=\tau+k+1}^{\infty} \delta^{j-(\tau+k)} f_z (z_{\tau T}^j, K_j)
\]

\[
> \left[ \sum_{j=\tau+k+1}^{\infty} \delta^{j-(\tau+k)} f_z (z_{\tau T}^{\tau+k}, K_{\tau+k}) \right],
\]

since \( z_{\tau T}^{\tau+k} > x_{\tau T}^{\tau+k}, K_j \geq K_{\tau+k}, j = \tau + k + 1, \ldots, T, \)

\[
= \delta f_z (x_{\tau T}^{\tau+k}, K_{\tau+k}) \frac{1}{1 - \delta}
\]

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i.e.,
\[ f_z(x_\tau^T, K_{\tau+k}) < (1 - \frac{1}{\delta}) \]
a contradiction.

**Example 23** In this example, assume and that \( f_z \) is strictly increasing in \( K \). Suppose that a firm invests strictly positive amounts in period 0 and in some other period - let \( t > 0 \) be the first such time period. For a firm entering in period \( \tau \)
\[ z^*(\tau) = \lim_{t \to \infty} z_t = x_\tau \]
where \( x_\tau \) is defined by
\[ 1 + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau} f_z(x_\tau, K_t) = 0 \]
We now claim that every entering firm earns strictly positive profit in the long run. Consider any period \( \bar{\tau} \geq 0 \) in which a strictly positive measure of firms enter. Using Proposition 9 there exists \( \bar{\tau} > \bar{\tau} \) such that entry occurs in period \( \bar{\tau} \) and further, we can choose \( \bar{\tau} \) such that \( K_{\bar{\tau}+1} > K_{\bar{\tau}+1} \). Since \( f_z \) is strictly increasing in \( K \) and \( z \), it follows that
\[ x_{\bar{\tau}} < x_{\bar{\tau}} \]
For if \( x_{\bar{\tau}} \geq x_{\bar{\tau}} \), then
\[ 0 = 1 + \sum_{t=\bar{\tau}+1}^{\infty} \delta^{t-\bar{\tau}} f_z(x_{\bar{\tau}}, K_t) \]
\[ \leq 1 + \sum_{t=\bar{\tau}+1}^{\infty} \delta^{t-\bar{\tau}} f_z(x_{\bar{\tau}}, K_t) \]
\[ = 1 + \sum_{t=\bar{\tau}+1}^{\infty} \delta^{t-\bar{\tau}} f_z(x_{\bar{\tau}}, K_{t-(\bar{\tau}-\bar{\tau})}) \]
\[ < 1 + \sum_{t=\bar{\tau}+1}^{\infty} \delta^{t-\bar{\tau}} f_z(x_{\bar{\tau}}, K_t) = 0 \]
that yields a contradiction. Thus, \( z^*(\bar{\tau}) > z^*(\bar{\tau}) \). Since (almost) no exit occurs in the industry, the limit price \( p^* \) and the limit industry capital stock \( K^* \) satisfies
\[ p^* \geq A(z^*(\bar{\tau}), K^*) \]
It follows that
\[ p^* > A(z^*(\bar{\tau}), K^*) \]
Thus, the limiting profit level for all firms that enter the market is strictly positive. In the limit, there are countably many different types of active firms in the industry, differentiated by their period of entry and holding different stocks of firm-specific capital and market profit, though they all produce the same output.
Example 24 Now, assume that the production cost is independent of industry-wide capital $K$ i.e., the fixed cost is just a function $f(z)$ of firm specific capital. From Proposition 9, we have that there is no entry in the industry after period 0. From Lemma 12, we have that no firm exits in finite time. Thus, all entry occurs in period 0 and these firms stay in the industry forever. Further, as discussed above, all investment by these firms takes place in period 0, $z_{0\infty}^0 = x_{0\infty}^0, \forall t \geq 1$. As no investment, entry or exit takes place from period 1 onwards, $p_t = p_{t+1}, \forall t \geq 1$. Let $p^*$ denote the constant level of price and $R^*$, the constant maximum profit every period from period 1 onwards. Then,

$$p^* > A(x_{0\infty}^0), R^* = \frac{1 - \delta}{\delta} x_{0\infty}^0 > 0$$

Thus, all firms have the same size and choose the same actions every period, there is no heterogeneity in the industry and all firms earn a strictly positive constant profit stream from period 1 onwards.

References


