1. Consider a duopoly with two identical firms whose cost function is given by:
\[ C(q) = q^2 \]
and the market demand is given by:
\[
D(p) = \begin{cases} 
1 - p, & 0 \leq p \leq 1 \\
0, & p > 1.
\end{cases}
\]

Derive the perfectly competitive or socially optimal outcome in this market. Next, suppose that firms set prices simultaneously and the demand for each firm is determined according to the efficient rationing rule. Show rigorously that both firms setting price at the perfectly competitive level is not a Nash equilibrium.

* Do you think there is a pure strategy NE? Explain.

2. Assuming that prices can only be quoted in cents, what do the reaction functions in the standard Bertrand model of price competition in a homogenous good market look like?

3. Do Review Exercise 22.


6. Consider a symmetric Cournot oligopoly in a homogenous good market with \( n \geq 1 \) firms where the cost function of each firm is given by:
\[ C(q) = \frac{q^2}{2} \]
and the market demand is given by
\[ D(p) = 1 - p. \]

(i) Derive the Cournot-Nash equilibrium and the associated price and profit. What happens to the industry outcome as \( n \to \infty \)?

(ii) Suppose that there is a fixed cost \( f > 0 \) of entering this market. There are an infinite number of potential firms (all identical) waiting to enter the market. Firms first simultaneously decide whether or not to enter the industry. Then, the firms that enter engage in Cournot quantity competition. Derive the equilibrium level of entry.

7. Consider a homogenous good market with a standard downward sloping demand curve. There is a fixed cost \( f > 0 \) of entering this market. There are an infinite number of potential firms waiting to enter the market. The cost function of each firm is given by \( C(q) = cq, 0 \leq c < P(0) \). Firms first simultaneously decide whether or not to enter the industry. Then, the firms that enter in stage
1. Engage in Bertrand price competition. Derive the equilibrium level of entry. Evaluate the equilibrium outcome from the point of social welfare.

8. Consider the same problem as in Question 7 change when there are two firms waiting to enter the industry, \( i = 1, 2 \). Each firm \( i \) produces at constant unit cost \( c_i \) where \( 0 \leq c_1 < c_2 < P(0) \). Is there an equilibrium (for some range of parameter values) where the high cost firm enters the industry and produces? Comment.

9. Consider a Cournot duopoly in a homogenous good market where the market demand is given by

\[
D(p) = \begin{cases} 
1 - p, & 0 \leq p \leq 1 \\
0, & p > 1.
\end{cases}
\]

Each firm \( i \) produces at constant unit cost \( c_i \) where \( 0 = c_1 < c_2 < 1 \). Derive the Cournot Nash equilibrium (for all \( c_2 \in (0, 1) \)) and evaluate the social welfare loss.

10. Go through Exercise 5.3. Now, suppose there are two firms in the market. Firm 1 now creates a division that is owned by firm 1 but runs and competes as an independent profit maximizing entity in the market (so that we have effectively three firms in the market). How does the joint profit of the two divisions of firm 1 now compare to its earlier profit? Comment on the economics of divisionalization.

11. Go through the example in Section 5.3.2.1. In this example, it is shown that (as the unit cost of capacity creation \( c_0 \in \left[\frac{3}{4}, 1\right] \)), capacity choice of each firm can be restricted \( ex \ ante \) to the interval \( \left[0, \frac{1}{4}\right] \). Show that this implies that capacities are in the set \( C \) defined more generally in the lecture notes.

If such restrictions on the capacity choice sets are not imposed \( ex \ ante \), one cannot rule out nonexistence of pure strategy NE. Provide an example of a pair of capacity levels that lead to non-existence of pure strategy NE in the second stage price subgame.

What is the equilibrium outcome of the two stage capacity price game if \( c_0 = 0 \)? What happens if \( c_0 = 0 \) but the unit production cost (set equal to zero in this section) is actually some number \( c \in \left[\frac{3}{4}, 1\right] \)?

Comment generally on the social welfare effect of publicly subsidizing capacity creation.