1. Consider a homogenous good industry with \( n \) identical firms where each firm has the cost function:

\[ C(q) = \frac{q^\alpha}{\alpha}, \alpha > 0. \]

Derive the industry’s total and marginal cost curve for the following values of \( \alpha \):

(i) \( \alpha = \frac{1}{2} \),
(ii) \( \alpha = 1 \),
(iii) \( \alpha = 2 \).

What is the effect of increase in \( n \) on the industry’s marginal cost curve in each case?

2. Suppose the industry in problem 1 has \( n = 10 \) firms and is inhabited by \( k = 5 \) identical consumers, each of whom has a quasi-linear utility function of the form:

\[ u = m_i + \ln x_i, m_i \in \mathbb{R}, x_i > 0. \]

where \( m_i \) is the consumption of a (non-produced) numeraire good and \( x_i \) is the consumption of the good produced by the industry. Assume there is a large enough endowment of the numeraire good and that \( C(q) \), as specified in problem 1, is the minimum amount of numeraire good required by a firm to produce \( q \) units of output. Derive the socially optimal level of output (approx) in cases (i)- (iii).

3. Consider a monopoly that faces market demand:

\[ D(p) = a - p, 0 \leq p \leq a \]
\[ = 0, p > a. \]

Derive the deadweight loss of monopoly and the Lerner index of market power if its cost function is given by

(i) \( C(q) = cq, 0 \leq c < a. \)
(ii) \( C(q) = \frac{q^2}{2}. \)

If a regulator knows the true cost function and wishes to attain efficiency through a output subsidy, what is the optimal subsidy?

4. Consider case (i) in problem 3. Suppose the regulator does not know the true value of \( c \) but knows that \( c = 0 \) with probability \( \frac{1}{2} \) and \( c = \frac{3}{4} \) with probability \( \frac{1}{2} \). Describe an optimal subsidy -price regulation scheme that can provide incentive for the firm to report truthfully and to produce the socially optimal level of output for both of the possible realizations of \( c \).
5. Consider a monopoly that produces two goods with interdependent demand and in each market $i$ faces the demand:

$$D_i(p_i, p_j) = a - bp_i + dp_j, d > 0, b > d, a > b - d$$

where it is understood that the demand is zero if the above expression is negative. Suppose that monopolist produces both goods at constant unit cost $c, 0 < c < \frac{a}{b-d}$. Derive the optimal prices in both markets and examine the effect of change in $d$ on market power in both markets. Interpret your answer.


8. Consider a monopolist who sells a durable good that lasts for two periods with no depreciation but becomes obsolete after 2 periods. The monopolist produces the durable good at constant unit cost set normalized to zero. There is a continuum of consumers (each consumer is non-atomic) of total mass = 2. Consumers have unit demand and are of two types - high valuation and low valuation, each of unit mass. The per period valuation (from using the durable good) is given by $V^H$ for high valuation consumers and $V^L$ for low valuation consumers where $V^H > 2V^L > 0$. Let $\delta \in (0, 1]$ be the common discount factor.

(i) Derive the optimal prices charged by a monopolist and her profit when she leases the good in each period.

(ii) Derive the optimal prices charged by a monopolist and her profit when she sells in both periods and (credibly) pre-commits in period 1 to the price in period 2.

(iii) Derive the optimal prices charged by a monopolist and her profit when she sells without being able to credibly pre-commit to the price in period 2.

(iv) Compare your answers above to that in the example of Bagnoli et al (1989) paper discussed in an example in class. Notice the similarity in structure and explain the difference in outcome.

(v) How does your answer to (i) and (iii) change in each of the following circumstances:

(a) In each period, the monopolist has the opportunity to exit this industry and use his talent/knowledge/capital in another industry where the per period profit is slightly higher than $V^L$.

(b) Unit cost of production in period 2 increases to a level slightly higher than $V^L$.

(c) The monopolist can, in case (iii), offer a money back guarantee that is credible (what kind of guarantee is he likely to offer?).

(d) $V^H > 3V^L$; a unit mass of new consumers with valuation $V^H$ and a unit mass of new consumers with valuation $V^L$ enter the market in period 2.