Lecture 1.
August 29, 2005.

Key points:
* 1950s-early 70s: Structure-Conduct-Performance paradigm (SCP) was the dominant theme.
* Structure = market concentration (also, product structure, technology, costs, vertical integration structure). Market concentration refers to how concentrated market shares are.
  * Market concentration = \( \frac{1}{n} \) in symmetric homogenous good market where \( n \) = number of firms.
  * More generally, concentration measured by Herfindahl index (H)

\[
H = \sum_{i=1}^{n} s_i^2
\]

\[
s_i = \frac{q_i}{Q} Q = \sum_{i=1}^{n} q_i
\]

\[
q_i = \text{output of firm } i, s_i = \text{market share of firm } i.
\]

Note that \( H \) reflects not only number of firms but the way market shares are distributed between firms.
* Conduct = behavior (pricing, R&D, investment, competitiveness, collusion, advertising...)
  * Performance refers to efficiency of market (profits, innovation, product variety, distributional consequence....)
  * In effect, Performance = market power (price markup above marginal cost).
  * Market power measured by Lerner index (L). In a monopoly,

\[
L = \frac{p - MC}{p}
\]

With more firms,

\[
L = \sum_{i=1}^{n} s_i \left[ \frac{p_i - MC_i}{p_i} \right]
\]

= (weighted) average of price-cost margin in the industry
= expected market power faced by an average consumer.
Note that if marginal cost for each firm $i$ is constant ($= c_i$), then

$$L = \sum_{i=1}^{n} s_i \left[ \frac{p_i - c_i}{p_i} \right]$$

$$= \sum_{i=1}^{n} s_i \left[ \frac{p_i q_i - c_i q_i}{p_i q_i} \right]$$

$$= \sum_{i=1}^{n} s_i \left[ \frac{\Pi_i}{R_i} \right], \Pi_i = \text{profit of firm } i, R_i = \text{revenue of firm } i,$$

$$= \text{(weighted) average profit rate in the industry.}$$

* SCP arose from public concern over market power.
* What explains market power? Market concentration? Empirical en-
query.
* Initially, assumed market power is bad. If discovered,identified, should
call for regulation or state intervention.
* Academic hypotheses of SCP paradigm:

$$S \rightarrow C :$$

greater concentration leads to higher collusion, more anti-competitive behavior
(entry barriers, predation etc.).

$$C \rightarrow P :$$

greater collusion, more anti-competitive behavior
leads to greater market power.

$$S \rightarrow P :$$

more concentration leads to higher prices.

$$\Rightarrow S \rightarrow P \text{ (by-pass Conduct).}$$

* Structure is primitive, performance is the explained variable.

*One of the primary forms of this hypotheses:

Higher market concentration (H) $\Rightarrow$ Higher market power (L).

* Almost mindlessly, ran cross section regressions (across industries) for over
20 years. Adding all kinds of variables.
* Measured L by profit rates.
* Net result: INCONCLUSIVE, AMBIGUOUS.
* DATA PROBLEMS. (profit rates often accounting profits).
* Endogeneity problem: S may be determined by C and P. May lead to
negative relationship between S and P.

Example: High profit rates may lead to high entry of new firms and, there-
fore, low concentration.
* Yielded no insight into conduct of firms (C treated as black box).
* The whole SCP enterprise was motivated by a premise that market power is bad. That is only true in a static perspective.
* In a dynamic world, industries that are technologically most progressive are likely to experience Schumpeterian creative destruction of old firms by new innovators who then yield lots of market power (the high price-cost margin compensating them for their investment in R&D and innovation) but are soon dispossessed by a fresh generation of leaders. At each point of time, a snapshot of this industry will show high concentration and high market power. But the identity of the monopoly power holders shifts radically over time. Consumers reap benefits of technological change. Industries that are technologically sedate on the other hand, exhibit none of these and consumers may actually be worse off there.
* Chicago School (Stigler etc.) brought out these kind of arguments in simple theoretical models. "Stop and think instead of mindlessly running regressions".
  * Existence of very high market power doubtful.
  * High concentration - even monopoly - may not lead to high market power. (For example, Coase conjecture in durable goods markets).
* One problem with early theory in I.O.: lack of tools to handle oligopolies.
* Early 1970s: non-cooperative game theory enters economic model building. Explosion of research in IO theory from mid-70s.
  * Directly analyzes Conduct instead of treating it as a black box and treats S,C,P as being simultaneously determined.
* Particularly, based on explicit analysis of strategic behavior in imperfectly competitive markets (Oligopolies), game theoretic foundations of observed behavior and institutions in markets; incomplete information.
  This course: Study of modern Industrial Organization theory.
* This theoretical explosion, has been followed by a new empirical IO literature that incorporates conduct of firms explicitly, uses firm-level disaggregated information and strategic behavior of firms and finally, looks at panel data on firms and industries to study dynamics.
  * A more recent wave of theory tries to catch up with the empirical findings here.
  * We won't go into the latter two.
Industry Cost Function.
Homogenous good industry with \( n \) incumbent firms.
Cost function of each firm \( j \): \( C_j(q_j) \).
Exclude sunk cost: \( C_j(0) = 0 \).
Assume \( C_j \geq 0 \), continuously differentiable and strictly increasing.
Suppose the industry produces total output \( Q > 0 \).
How should production be organized in the industry so as to minimize the total industry wide production cost?

\[
\min \sum_{j=1}^{n} C_j(q_j) \quad (1)
\]

subject to
\[
\sum_{j=1}^{n} q_j = Q, q_j \geq 0, j = 1, \ldots, n.
\]

First order condition: In any industry cost minimizing solution
\[
q_j > 0, q_k > 0 \Rightarrow C'_j(q_j) = C'_k(q_k)
\]

Equi-marginal cost principle: To minimize industry’s production cost, all firms that produce positive output must be equating their marginal cost.

Let \( \Phi(Q) \) denote the minimum industry cost of producing \( Q \).
Assume \( \Phi(Q) \) is differentiable. Then (using the envelope theorem),
\[
\Phi'(Q) = C'_j(q_j)
\]

where \( j \) is a firm that produces strictly positive level of output in the industry cost minimizing solution. Thus, the industry’s marginal cost of production is equal to the marginal cost of each firm that produces positive output.

Observe:
1. If all firms are characterized by increasing returns to scale i.e., for all firms \( j \), \( AC_j(q) = \frac{C_j(q)}{q} \) is strictly decreasing in \( q \), then industry cost is minimized by concentrating all production in 1 firm (viz., the firm where \( AC_j(Q) \) is smallest).

   This remains true even if the AC curve of each firm is not globally decreasing but declines in the relevant range (between 0 and the point where the average cost curve intersects the market demand curve for the last time).

   This is called a situation of natural monopoly.

2. If all firms are characterized by constant returns to scale, then concentrating all production in one firm (where the unit cost is lowest) is a solution. If firms are identical, any division of output between the firms is optimal.

3. If all firms are characterized by decreasing returns to scale i.e., for all firms \( j \), \( AC_j(q) = \frac{C_j(q)}{q} \) is strictly increasing in \( q \), then it is likely that industry cost is minimized by dividing production between several firms. In particular, if all firms are identical, then each firm produces \( \frac{Q}{n} \).

Try this on your own: Prove # 3.
Assume that $\forall j = 1, \ldots, n, C_j(q)$ is convex on $\mathbb{R}_+$. It can be shown that (try it) this implies:

a) non-increasing returns to scale i.e., $AC_j(q)$ is non-decreasing in $q$. 

b) marginal cost $C'_j(q)$ of each firm is non-decreasing in $q$.

The industry's marginal cost curve can be derived by a horizontal aggregation of the individual firm's marginal cost curves.

Remark: The analysis above has to be suitably modified when firms are not incumbent and there is a fixed cost of entry. Of course, in markets with differentiated goods and in markets where firms make other decisions (such as investment in R&D, capital, advertising) that affect cost and demand, the analysis is far more complicated.

**Partial Equilibrium Welfare Analysis: A simple framework.**

An economy with two goods: good $l$ and a numeraire good $m$. Good $l$ is produced but good $m$ is non-produced (endowment).

Think of good $m$ as a composite commodity: "expenditure on all other goods".

$H$ consumers or agents.

Let $x_i$ and $m_i$ denote consumer $i$'s consumption of good $l$ and the numeraire. Each consumer $i = 1, \ldots, H$ has a utility function

$$u_i(x_i, m_i) = m_i + \phi_i(x_i), x_i \geq 0, m_i \in \mathbb{R}$$

Allowing negative consumption of the numeraire good is simply a way to avoid keeping track of too many "corner solutions". But one can easily extend the analysis to the case where we restrict $m_i$ to be non-negative.

Assume: $\phi_i(\cdot)$ is twice differentiable, $\phi_i(0) = 0$, $\phi'_i(x) > 0, \phi''_i(x) < 0$ at all $x \geq 0$.

There are $n$ firms. Each firm $j$ can produce good $l$ by using good $m$. In particular, the amount of good $m$ required by firm $j$ to produce $q_j$ amount of good $l$ is given by $C_j(q_j)$; this is firm $j$'s cost function.

Assume: $C_j(q)$ is twice differentiable, $C'_j(q) > 0, C''_j(q) \geq 0$ at all $q \geq 0$.

Total endowment of good $m$ in the economy: $\omega_m$.

No initial endowment of good $l$.

A feasible allocation in this economy is a vector $((x_1, m_1), \ldots, (x_H, m_H), q_1, \ldots, q_n)$ such that

$$\sum_{i=1}^{H} x_i \leq \sum_{j=1}^{n} q_j$$

$$\sum_{i=1}^{H} m_i + \sum_{j=1}^{n} C_j(q_j) \leq \omega_m$$

$$x_i \geq 0, q_j \geq 0.$$

Each such allocation generates a certain profile of utility $\{u_i(x_i, m_i)\}_{i=1}^{H}$ for the consumers or agents.
A feasible allocation is Pareto efficient (or Pareto optimal) if there does not exist any other feasible allocation that generates at least as much utility for all agents and strictly higher utility for at least one agent.

Suppose we fix the consumption and production vectors of good \( l \) at some arbitrary levels \((\pi_1, \ldots, \pi_H, \pi_1, \ldots, \pi_n) \geq 0\). The total amount of the numeraire good available for distribution to consumers is \([\omega - \sum_{j=1}^{n} C_j(\pi_j)])\). The set of utilities that can be generated by varying the distribution of this amount of numeraire good:

\[
\{(u_1, \ldots, u_H) : \sum_{i=1}^{H} u_i \leq \sum_{i=1}^{H} \phi_i(\pi_i) + [\omega - \sum_{j=1}^{n} C_j(\pi_j)]\}
\]

This is a set whose boundary, the utility possibility frontier given \((\pi_1, \ldots, \pi_H, \pi_1, \ldots, \pi_n)\), is a hyperplane whose normal is the 45-degree line. One way to see this is to understand that given \((\pi_1, \ldots, \pi_H, \pi_1, \ldots, \pi_n)\), the right hand side of the above inequality is a constant.

If we alter the fixed consumption and production vectors of good \( l \) \((x_1^*, \ldots, x_H^*, q_1^*, \ldots, q_n^*)\), we can move in a parallel fashion to a different utility possibility frontier (depending on whether the right hand side of the above inequality increases or decreases).

A Pareto efficient allocation must be one where the consumption and production vectors of good \( l \) \((x_1^*, \ldots, x_H^*, q_1^*, \ldots, q_n^*)\) are such that the highest possible utility possibility frontier is attained i.e., the right hand side of the above inequality is maximized.

In other words, a feasible allocation \(((x_1^*, m_1^*), \ldots, (x_H^*, m_H^*), q_1^*, \ldots, q_n^*)\) is Pareto efficient if and only if \((x_1^*, \ldots, x_H^*, q_1^*, \ldots, q_n^*)\) solves:

\[
\max \left[ \sum_{i=1}^{H} \phi_i(x_i) - \sum_{j=1}^{n} C_j(q_j) \right] \quad (2)
\]

s.t. \( \sum_{i=1}^{H} x_i \leq \sum_{j=1}^{n} q_j, x_i \geq 0, q_j \geq 0. \)

We call the maximand \([\sum_{i=1}^{H} \phi_i(x_i) - \sum_{j=1}^{n} C_j(q_j)]\) the Marshallian (net) social surplus.

In a quasi-linear economy, Pareto efficiency becomes equivalent to social welfare maximization where the welfare function is the Marshallian net social surplus.

In any solution to the above maximization problem, \( \sum_{i=1}^{H} x_i = \sum_{j=1}^{n} q_j \).

One can break-down this maximization problem as follows:
For each $x \geq 0$, let $f(x)$ be defined by

$$f(x) = \max \sum_{i=1}^{H} \phi_i(x_i) \quad (3)$$

s.t. $\sum_{i=1}^{H} x_i = x, x_i \geq 0$.

Then, $f(x)$ is the maximum utility generated by consumption of $x$ units of good $l$ in society.

Recall, the definition of industry cost $\Phi(.)$.

Then first solve

$$\max_{Q \geq 0} [f(Q) - \Phi(Q)] \quad (4)$$

If $Q^*$ solves this problem, set $(x^*_1, \ldots, x^*_H)$ equal to a solution to (3) for $x = Q^*$ and $(q^*_1, \ldots, q^*_n)$ equal to a solution to (1) for $Q = Q^*$. From the first order condition,

$$\phi'_i(x_i) = \phi'_k(x_k)$$

for any $i, k$ such that $x_i > 0, x_k > 0$ is a solution to (3). Suppose $H'$ agents consume strictly positive quantity. Let $P(x)$ denote the common marginal utility $\phi'_i(x_i)$ of all such agents. Then,

$$f(Q) = \int_0^Q df(x)$$

$$= \int_0^Q d(\sum_{i=1}^{H'} \phi_i(x_i))$$

$$= \int_0^Q \sum_{i=1}^{H'} \phi'_i(x_i) dx_i$$

$$= \int_0^Q P(x) \left( \sum_{i=1}^{H'} dx_i \right)$$

$$= \int_0^Q P(x) dx$$

Thus, the maximization problem in (4), reduces to

$$\max_{Q \geq 0}[\int_0^Q P(x)dx - \Phi(Q)] \quad (5)$$

Observe that, if we set up the utility maximization problem of agent $i$ subject to a budget constraint (leaving out some details here), then the first order condition for an interior maximum:

$$\phi'_i(x_i) = p$$
where \( p \) is the price of good \( l \). \( \phi'_i(x_i) \) is the inverse-demand function of agent \( i \). Further, verify that, \( P(x) \) is the inverse-market demand function, derived by horizontal summation of individual inverse demand functions.

\[
\int_0^Q P(x) \, dx
\]

is the area under the market demand curve and this is equal to the sum of the areas under the individual demand curves \( \phi'_i() \) between 0 and the quantity \( x_i \) consumed by agent \( i \) in any solution to (3).

We interpret this as society’s (maximum) gross surplus, benefit or the (total) willingness to pay for quantity \( Q \). When we subtract from this the industry’s (minimum) total cost of production, we obtain the net social surplus from quantity \( Q \) which is the maximand in (5). The latter can also be written as

\[
\int_0^Q P(x) \, dx - P(Q)Q + [P(Q)Q - \Phi(Q)]
\]

Consumers Surplus + Producers Surplus

when the output \( Q \) is sold at demand price \( P(Q) \).

First order necessary condition for maximization problem (5)

\[
P(Q^*) = \Phi'(Q^*)
\]

Thus, Pareto efficiency requires that the industry produce an output such that the demand price (society’s marginal willingness to pay or marginal benefit) is exactly equal to the industry’s marginal cost. Further, the output must be distributed across firms in a manner so as to minimize industry’s production cost. This implies that the demand price must equal the marginal cost of each firm that produces positive output. [It must also equal the marginal utility or marginal willingness to pay of each consumer].

A perfectly competitive equilibrium, if it exists, will satisfy all of these conditions (the industry’s marginal cost curve is nothing but the aggregate supply curve in such a market).

This characterization of efficiency breaks down if there are production or consumption externalities.
Monopoly.
Homogenous good market and simple pricing.
Cost function: $C(q)$.
Market demand function: $D(p)$; Inverse demand: $P(q)$.
* Market power:

$$
L = \frac{1}{\eta(p^m)}
$$

where $\eta(p)$ is the price elasticity of demand at price $p$ and $p^m$ is the monopoly price.

Monopoly power in the sense of deviation from marginal cost pricing is higher in markets with less elastic demand.

* Deadweight loss of monopoly:

$$
DWL = \int_{q^m}^{q^c} [P(q) - C'(q)]dq
$$

where $q^c$ is the socially optimal output defined by

$$
P(q) = C'(q^c).
$$

DWL also called welfare loss (efficiency loss) due to monopoly and the Harberger triangle.

* DWL is actually an upper bound on efficiency gain possible through state intervention in a monopoly market - the actual gains might be much smaller.

* DWL does not necessarily decrease with price elasticity of demand.

* Empirical estimates of DWL: very small. Harberger (1954): 0.1% of GDP. Controversial.

* Policy measures to correct DWL in monopolies:
  a) Subsidy (not tax): Output subsidy $s$ per unit. Optimal subsidy:

$$
S = -q^cP'(q^c)
$$

Subsidy induces consumers to consume more by reducing the effective price faced by them.

Problem 1: Financing subsidy through taxes might cause other distortions and welfare loss in the economy.

Problem 2: Setting subsidy optimally requires that regulator have information on cost function - often a private information of the firm. Firm has incentive to secure higher subsidy by mis-reporting cost.

Problem 3: Regulatory capture.

b) Price regulation (price ceiling).

Problems 2 and 3 exist here too.
* Additional problem with price regulation: in natural monopoly (increasing returns to scale), marginal cost pricing implies firm earns loss. Need subsidy.

Way out: set price regulation such that price = average cost. (Called Rate of return regulation: "fair" rate of return on capital).
Firm breaks even.
Not fully efficient but reduces DWL.
Problem: Creates horrible incentives for cost reduction, R&D, quality improvement etc.
Way out: Lagged price adjustment so that firms can earn positive rent for some time.

* Current regulation literature: Asymmetric information between firm and regulator is the most serious problem in regulation of monopoly. Particularly, when firm’s own information is imprecise.

Simple example: Constant returns to scale; Constant unit cost = \( c \).
Regulator does not know \( c = c_L \) or \( c_H \).

\[ 0 < c_L < c_H \]

Firm knows its unit cost.
Regulator asks firm to report unit cost and sets a price ceiling \( p(c) \) according to the reported unit cost.
Assume: monopoly price corresponding to unit cost \( c_L > c_H \).
If \( p(c_L) \neq p(c_H) \), firm will report the value of unit cost that leads to higher price ceiling.
Cannot secure both truthful reporting and efficiency with simple price regulation.
For example, efficiency requires \( p(c_H) = c_H, p(c_L) = c_L \), but then monopolist will lie if actual cost is \( c_L \).
Need subsidy. For example, a lump sum subsidy \( A \) to be paid if reported unit cost is \( c_L \). Set \( p(c_H) = c_H, p(c_L) = c_L \).

\[ A = (c_H - c_L)D(c_H) \]

In general, monopolist must be paid informational rent for truthful reporting - lower his cost, higher the rent paid.
But such rents increase the incentive for regulatory capture and need to be financed externally.

* Broad economic view: Except in the case of natural monopoly, best way to deal with other monopolies is to address why there is no greater competition in the market - for example, remove barriers to entry and incentives for predation of small new entrants by judicious review of the conduct of monopoly firm; remove barriers to international trade so as to introduce foreign competition etc.

* Other sources of welfare loss due to monopoly power:
1. Rent seeking behavior.
(Welfare loss may include DWL + monopoly profit; may even exceed it).
2. Cost Distortion due to lack of yardstick competition.
Lecture 4
Key points.

* Multiproduct monopoly: $n$ markets, possibly interdependent demand & costs.

$$\max_p \left\{ \sum_{i=1}^n p_i D_i(p) \right\} - C(D_1(p), ..., D_n(p))$$

* Independent demand, dependent costs.

1. Cost complementarity $\Rightarrow$ increased output of one good reduces cost of production of other goods
   $\Rightarrow$ lower price and higher output compared to the level at which $p_i - MC_i = \frac{1}{\eta_i}$
   $\Rightarrow$ lower market power observed than one good simple monopoly case.

Ex. economies of scope in joint production. For $n = 2$,

$$C(q_1, q_2) = C_1(q_1, q_2) + C_2(q_1, q_2), \frac{\partial C_i}{\partial q_j} < 0.$$ 

Ex. (slightly different) Learning by doing - interpret different goods as the same market in different time periods. Increased output today reduces tomorrow’s cost through greater experience. Two period case:

$$C(q_1, q_2) = C_1(q_1) + \delta C_2(q_1, q_2), \frac{\partial C_2}{\partial q_1} < 0$$

$$\max_{q_1, q_2} \left[ \left( P(q_1)q_1 + \delta P(q_2)q_2 - C_1(q_1) - \delta C_2(q_1, q_2) \right) \right]$$

where $\delta$ is the discount factor.

2. Cost congestion: increased output of one good may raise cost of production for other good. Reverse effect.

* Dependent demand, independent cost.

$$\max_p \left\{ \sum_{i=1}^n p_i D_i(p) - C_i(D_i(p)) \right\}$$

First order condition:

$$L_i = \frac{p_i - MC_i}{p_i} = \frac{1}{\eta_{ii}} \left[ 1 + \sum_{j \neq i} \frac{p_i}{D_j} \frac{\partial D_j}{\partial p_i} \right]$$

where $\eta_{ii}$ is own-price elasticity of good $i$.

Substitute goods: $\frac{\partial D_j}{\partial p_i} > 0$, $L_i > \frac{1}{\eta_{ii}}$.

Complements: $\frac{\partial D_j}{\partial p_i} < 0$, $L_i < \frac{1}{\eta_{ii}}$.

Ex. Goodwill in intertemporal pricing.

$$\max_{p_1, p_2} D_1(p_1) + \delta D_2(p_2, p_1) - C_1(D_1(p_1)) - \delta C_2(D_2(p_2, p_1))$$
where
\[ \frac{\partial D_2}{\partial p_1} < 0. \]

Low market power in period 1.
Ex. Repeat purchase. Customers more likely to buy tomorrow, if they buy today.

**DURABLE GOOD MONOPOLY.**
Durable goods last for more than one period.
Demand is linked over time.
Suppose a monopolist has the flexibility to charge different prices over time (intertemporal price discrimination).
Downward sloping market demand curve for the durable good with price taking consumers (for example, continuum of consumers).
As a monopolist, if you sell a lot today, you reduce the demand for your own good tomorrow.
If the demand is lower tomorrow, you will have incentive to reduce your price tomorrow.

Consumers wait to buy if they believe prices will be significantly lower later (or quality higher). Incentive to wait depends on price difference perceived and the length of time interval between price revisions.
As a monopolist, you are competing with future versions of yourself.

*Coase conjecture* [Coase (J. Law Economics & Organization, 1972)]: The flexibility afforded by the ability to charge different prices over time will hurt the monopolist. Consumers will tend to wait for lower prices in the future. This will force the monopolist to charge lower prices today. As the time interval between successive price revisions goes to zero, monopolist loses all market power. In the limit, you have a competitive outcome.

The monopolist is much better off if he can credibly pre-commit to fix price at the initial level forever.
The monopolist can also do better if he leases the good instead of selling it. Leasing converts a market for a durable good into a market for the current service from the good which is, by definition, non-durable.

A simple two period model due to Bulow (J. Political Economy, 1982):
\[ t = 1, 2. \]
Good bought in period 1 lasts for two periods with no depreciation. After period 2, it becomes obsolete.
Assume production cost = 0.
Demand for use of the good each period: \( D(p) = 1 - p. \)
[Think of this as if there is a unit mass of consumers who use at most one unit of this good each period and their dollar valuation of this use per period is distributed uniformly between 0 and 1.]
Leasing:
No link between periods.
Monopolist sets $p_t$ so as
\[
\max_{p_t} p_t (1 - p_t)
\]
i.e., $p_1 = p_2 = \frac{1}{2}$ and the total discounted sum of profits is $\frac{1}{4}(1 + \delta)$.

Selling with pre-commitment to not lower price in period 2:
Consumers who buy do so in period 1.
If quantity $q$ is sold then marginal consumer’s use valuation must equal the price charged:
\[
p = (1 + \delta)(1 - q)
\]
and so monopolist solves
\[
\max_q q(1 + \delta)(1 - q)
\]
which yields, $q_1 = \frac{1}{2}, p_1 = \frac{(1 + \delta)}{2}$ and the total discounted sum of profits is $\frac{1}{4}(1 + \delta)$.

Selling (no pre-commitment):
Work backwards.
Now, there is a (second hand) resale market in period 2. But there is no physical difference between resold and new good in period 2.
So, if $q_1 (\leq 1)$ is the quantity of goods sold by the monopolist in period 1 and $q_2$ is the quantity of new goods sold in period 2, then the price in period 2 must satisfy:
\[
p_2 = 1 - (q_1 + q_2)
\]
So, for any $q_1$, in period 2, the monopolist will set $q_2$ to maximize $[1 - (q_1 + q_2)]q_2$ which yields:
\[
q_2(q_1) = \frac{1 - q_1}{2} = p_2(q_1)
\]
and the profit in period 2 is
\[
\pi_2(q_1) = \frac{(1 - q_1)^2}{4}
\]
Now, we go to period 1. If monopolist sells $q_1$ in period 1, the highest price at which he can sell it will be a price $p_1(q_1)$ such that the marginal consumer (the lowest valuation consumer that buys) i.e., the one with utilization valuation equal to $(1 - q_1)$ is indifferent between buying the good in period 1 (and enjoying it for two periods) or waiting to buy it in period 2 at a price $p_2(q_1)$:
\[
(1 - q_1)(1 + \delta) - p_1(q_1) = \delta(1 - q_1 - p_2(q_1))
\]
\[
= \delta\left(\frac{1 - q_1}{2}\right)
\]
so that
\[
p_1(q_1) = (1 + \frac{\delta}{2})(1 - q_1)
\]
and the intertemporal discounted sum of profits to the monopolist by selling $q_1$ in period 1:

$$q_1 p_1(q_1) + \delta \pi_2(q_1) = (1 + \frac{\delta}{2})q_1(1 - q_1) + \delta \frac{(1 - q_1)^2}{4}$$

and maximizing this yields

$$q_1^* = \frac{2}{4 + \delta}$$

so that

$$p_1^* = \frac{(2 + \delta)^2}{2(4 + \delta)} < \frac{(1 + \delta)}{2}$$

and the total discounted sum of profits:

$$[1 - \frac{\delta}{4}] \frac{2 + \delta}{4 + \delta}^2$$

which is strictly less than $\frac{1}{4}(1 + \delta)$, the total discounted sum of profits when the monopolist leases or sells with pre-commitment to not lower prices.

Formal version of the Coase Conjecture:

Infinitely lived monopolist with unit cost of production $c \geq 0$.

Assume there is a continuum of infinitely lived consumers with unit demand whose valuation of the durable good (the infinite horizon discounted sum of use value) is distributed on $[c, \infty)$

Prices revised at points of time of interval $\Delta > 0$. Discount factor:

$$\delta = e^{-r\Delta}$$

where $r$ is the interest rate.

Result: As $\Delta \rightarrow 0$, the intertemporal (discounted sum of) profit of the monopolist $\rightarrow 0$ and all prices converge to $c$. In the limit, almost all trades take place at the initial instant.

Main arguments:
Continuum of consumers implies consumers are price taking. They decide according to their expectation of future prices which they take as given.
Rational expectations equilibrium: consumers anticipate the price path to be chosen by monopolist, take this as given and buy accordingly; given this
buying behavior, there should no incentive for the monopolist to deviate from
the price path at any point.

Incentive to wait is lower for higher valuation consumers. For example, the
gain from buying today rather than tomorrow:

\[
(v - p_t) - \delta(v - p_{t+1}) = (1 - \delta)v - p_t + \delta p_{t+1}
\]

is increasing in \(v\).

If consumer with a certain valuation buys today, all consumers with higher
valuation must have bought by the end of today.

For any fixed \(\Delta > 0\), equilibrium price path is non-increasing over time. If
price increases tomorrow, no one will buy tomorrow.

As \(\Delta\) becomes extremely small, buyers will wait even if the price declines
very slightly and so the price decline must be extremely small if you want some
consumers to buy today rather than wait.

As \(\Delta \to 0\), gain from intertemporal price discrimination becomes negligible.
WARNING: Actual proof is much more complicated.

Note: The durable goods monopoly problem is equivalent to a problem of
bargaining under one sided incomplete information.

* In general, monopolist can do much better by precommitting to a sequence
of prices (the optimal precommitment turns out to be a constant price sequence
equal to the static monopoly price). The latter also generates same profit as
the leasing outcome.

* Coase Conjecture does not hold with finite number of consumers (discrete
demand) who behave strategically.

Lecture 5
Key points.

Durable Goods Monopoly (Contd).

* Coase Conjecture does not hold with finite number of consumers (discrete demand) who behave strategically.


  Main argument: higher valuation buyers realize how their buying decision influences future prices and that if they wait to buy at a lower price, the monopolist will not have incentive to charge lower price (until they have bought and left the market).

  Example.

  Consider a durable good that lasts two periods, \(t = 1, 2\).

  No depreciation, but good becomes obsolete after period 2.

  Consumers have unit demand (i.e., buy at most one unit).

  There are two consumers: a high valuation consumer whose valuation of (maximum willingness to pay for or gross surplus from) one period’s use of one unit is \(V_H\) and low valuation consumer whose valuation of one period’s use of one unit is \(V_L\).

  Assume: \(V_H > 2V_L\).

  Assume cost of production = 0.

  Leasing outcome:

  Monopolist charges \(p = V_H\) every period earning profit \(\pi^L = (1 + \delta)V_H\).

  Selling with pre-commitment about price in period 2.

  Here, monopolist has to pre-commit to \(p_1, p_2\) in period 1 and cannot condition \(p_2\) on what happens in period 1.

  Monopolist charges \(p_1 = (1 + \delta)V_H\) and pre-commits to \(p_2 \geq V_H\) earning profit \(\pi^{SPC} = (1 + \delta)V_H\).

  [Note if monopolist wants to sell to both consumers in period 1, his profit is \(2(1 + \delta)V_L < (1 + \delta)V_H\). If he plans to sell only to the high valuation consumer in period 1 and sell to the low valuation consumer in period 2, then (check that) he will charge \(V_L\) in period 2 (charging a lower price in period 2 only reduces his profit and increases the incentive of the high valuation buyer to wait for period 2) and in period 1, he will charge a price \(p_1\) so as to leave the high valuation buyer indifferent between buying in periods 1 and 2:

  \[(1 + \delta)V_H - p_1 = \delta(V_H - V_L)\]

  so that

  \[p_1 = V_H + \delta V_L\]

  and the profit of the monopolist is \(V_H + 2\delta V_L < (1 + \delta)V_H\].]

  Selling without pre-commitment.

  This is a two stage game with three players. One has to look for a subgame perfect equilibrium (SPE).
Strategy in period 2: specifies an action to be chosen for each possible history of the play (i.e., what happened in period 1).

Consider the following strategies:

Monopolist: I will charge \( p_1 = (1 + \delta)V^H \) in period 1; in period 2, I will charge a price equal to the highest valuation of the buyers remaining in the market.

Buyer of type \( i \): I will buy in the first period if \( p_1 \leq (1 + \delta)V^i, i = H, L \). If not, I will wait for period 2 and buy if \( p_2 \leq V^i \).

Claim: These strategies constitute a SPE.

Proof: Work backwards.

First consider period 2 and check that no player has an incentive to deviate. Given the buyers’ buying strategy, for the monopolist, its optimal to charge \( p_2 = V^H \) if the high valuation buyer is left in the market (with or without the other buyer) and to charge \( p_2 = V^L \), if only the low valuation buyer is left. So, the above mentioned strategy is clearly optimal for the monopolist in stage 2. For each buyer that has not bought in period 1, it is optimal to buy as long as the price does not exceed their one period use value. So, buyers’ strategy is optimal too.

Now, go back to period 1. Each buyer knows that given the strategy of the monopolist, she can get at most zero net surplus in period 2 i.e., either \( p_2 \) will exceed her valuation or will exactly equal her valuation. So, if \( p_1 \leq (1 + \delta)V^i \), it is optimal for a buyer with valuation \( V^i \) to buy in period 1. (Note: this is the critical argument, the high valuation buyer cannot reduce the price at which she buys by waiting). Finally, given the strategies of the buyers, the monopolist is clearly better off (check this) charging \( p_1 = (1 + \delta)V^H \) as the strategy of high valuation buyer says she is going to buy at that price in period 1 and, by backward induction, monopolist knows she can sell to low valuation buyer next period at price \( p_2 = V^L \). QED.

When this equilibrium is played, the monopolist price discriminates over time (and therefore, across consumers) and the profit is

\[
\pi^S = (1 + \delta)V^H + \delta V^L.
\]

Note:

\[
\pi^S > \pi^{SPC} = \pi^L = (1 + \delta)V^H
\]

Thus, the Coasian problem disappears. Selling is better than leasing. Even without pre-commitment, monopolist makes much higher profit than leasing. Pre-commitment to future prices not better for monopolist.

More generally, it is shown that with finite number of consumers, there are equilibria where the Coase conjecture does not hold. Indeed, in the infinite horizon case, there are equilibria such that as discount factor goes to one, the monopolist exercises almost perfect market power extracting almost all social surplus.
Other settings in which the Coase conjecture (and related results) may not hold:

* Decreasing returns to scale (Kahn (Econometrica, 1986)) i.e., upward sloping MC curve can convince consumers that prices cannot fall very fast (monopoly output cannot expand without raising MC that in turn acts as a floor on pricing).

* Fixed opportunity cost of staying in the market - monopolist exits if remaining demand is not high enough - so price cannot fall too much in the future.

* Strong depreciation.

* Increasing marginal cost over time (cost congestion for example, when production over time involves use of a finite stock of necessary input such as an exhaustible natural resource).

* Asymmetric Information: buyers may not know MC of seller. Low cost seller can pretend to be a high cost seller, charge a price equal to MC of high cost seller and earn positive profit that is bounded away from zero (even if interval between price revisions are close to zero).

* Inflow of new cohorts of buyers (Conlisk, Gerstner and Sobel, Quarterly J. of Econ, 1984) - identical cohorts enter over time - lower valuation buyers have higher incentive to wait. So, stock of consumers waiting to buy are on the average of lower valuation than new cohort. Monopolist has incentive to not lower price too much and sell to higher valuation new consumers, until the stock of old lower valuation consumers become large and at that point to have a big sale which, in turn, reduces the stock of consumers sharply and then return to high prices again. Price cycles.

Other mitigating factors that allow durable good sellers to make money:

* Pre-commitment (third party).

* Leasing

* Reputation: If consumers and seller are infinitely lived, the monopolist may establish a reputation for not cutting prices. Consumers expect monopolist to be "honorable" & not to cut prices and if he does so in any period, they expect that he is actually a "scoundrel rational seller" i.e., one who will cut prices if it suits him and therefore, the Coase conjecture works from that point on and drives his profit to near zero. Therefore, even though a rational monopolist might want to cut prices, he may find he is better off pretending to be an honorable guy and not cut prices (as that might spoil his reputation & will make him earn close to zero profit from that point onwards).

* Money back guarantee - compensate consumers for all future price declines. Effectively, monopolist pre-commits to not cut price. Problems in enforcement (secret price cuts, quality improvement etc).

* Planned Obsolescence: Reducing durability reduces the quantity of goods carried over and thus convinces buyers that the price won’t fall too much. Textbook producers frequently bring out new editions to kill used books. Got to be careful: consumers anticipate the degree of planned obsolescence and their
valuation falls; this forces seller to reduce the level of initial price. (see, for example, Waldman (Quarterly J. of Economics, 1993).

* Oligopolistic collusion: The same mechanism that allows oligopolists to collude may also allow them to pre-commit to not cutting prices rapidly over time

REMARK: Coasian problems arise also when seller can innovate and improve product over time.