Buyer Heterogeneity and Dynamic Sorting in Markets for Durable Lemons

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Abstract

In a durable good market where sellers have private information about quality, I identify certain problems in dynamic trading and sorting that arise in the presence of heterogeneity among buyers. Higher valuation buyers may have lower incentive to wait to buy better quality at a later time period. As a result, even though higher quality sellers may wait to sell later at higher prices and distinguish themselves from lower quality sellers, the dynamic price mechanism may not be effective as a sorting device. Under certain conditions, the range of quality traded (over all time periods) may be limited, sometimes no more than that in the static case. Further, even when all qualities are eventually traded, higher valuation buyers may consume lower quality goods, leading to inefficiency in allocation in addition to that caused by delay in trading.

Key-words: Asymmetric Information, Adverse Selection, Durable Good, Dynamic Trading, Dynamic Sorting, Heterogenous Buyers.

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1 Introduction

A fundamental problem of resource allocation in markets relates to adverse selection arising from asymmetric information among traders. In a static model of a competitive market where traders care about quality, and quality is observed by sellers but not by potential buyers, Akerlof (1970) and Wilson (1980) showed that the market outcome may involve no trading or, at best, trading of only low quality goods. Subsequently, a very large literature has examined how the market may overcome these problems when agents have incentives to choose costly actions that modify the information structure through signaling and screening mechanisms.

Most of the standard examples of markets with adverse selection involve durable goods where potential sellers are current owners of the goods that acquire private information about the quality of the good. Over the last decade, a growing literature has studied the manner in which the dynamic nature of such durable goods markets may endogenously resolve some of the trading problems encountered in static models. This paper is part of a strand of this literature which takes a given asymmetric information structure among traders in a single market, and examines how repeated trading opportunities and differential cost of waiting can lead to expanded trading through intertemporal sorting of privately informed traders even when there are no other possibilities of signalling or screening of private information.\(^1\) Thus, Janssen and Roy (2002) focus on the dynamic competitive price mechanism in a durable good market in a model that is a direct extension of the Akerlof-Wilson static model to an infinite horizon setting. Assuming that buyers are homogeneous, that the range of quality is bounded and that no entry of traders occurs in this market after the initial period, they show that all goods are traded in finite time; the equilibrium path involves better quality being traded at higher prices over time.\(^2\) The only source of inefficiency in the market is the cost of waiting, and the extent of waiting on the equilibrium path is related to impatience. Similar qualitative results have also been derived in markets with search frictions (Inderst and Müller, 2002) and decentralized trading with trading frictions (House and Leahy, 2004, Moreno and

\(^1\) Another strand of this literature has emphasized the fact that the ownership of goods, information structure and incentives of traders in used goods markets are endogenously determined through sequential interaction among primary and various secondary goods market, and that this can mitigate the "lemons" problem (Hendel and Lizzeri, 1999). Hendel, Lizzeri and Siniscalchi (2004) show that with distinct rental markets for secondary goods of differing vintages, the number of times a good has changed hands may accurately signal quality and lead to first best efficiency.

\(^2\) This result has been extended to models with entry of cohorts of sellers over time (Janssen and Karamychev, 2002, Janssen and Roy, 2004).
Wooders, 2010, Kim, 2011). The common assumption in this strand of the literature is that potential buyers are homogenous. The purpose of this note is to illustrate some potential problems in dynamic trading that can emerge when we allow for *heterogeneity of buyers*.

I use a simple discrete version of the model in Janssen and Roy (2002) with two qualities (high and low) and two types of buyers. Higher valuation buyers may have lower incentive to wait and buy better quality at a later time period. As a result, even if higher quality sellers have greater incentive to sell later at higher prices, the dynamic price mechanism may not be effective as a sorting device. Under certain conditions, only low quality goods may be traded (as in the static model). Further, even when the market outcome is one where all qualities are eventually traded, high valuation buyers may consume low quality goods, leading to inefficiency in allocation in addition to that caused by delay in trading.

The next section outlines the framework and some benchmark results. Section 3 contains results on impossibility of trading high quality goods. Section 4 discusses inefficiency in allocation under dynamic trading arising from high valuation buyers buying low quality goods. Section 5 concludes.

## 2 Framework

Consider a dynamic competitive market for a perfectly durable good\(^5\). Time is discrete and denoted by \(t = 1, 2, \ldots, \infty\). There is a continuum of price taking sellers and buyers that are infinitely lived. All traders enter the market in the initial period and no entry occurs after that. Traders leave the market immediately after trading.\(^6\) All traders are risk neutral and discount the future using a uniform discount factor \(\delta \in (0, 1)\). The valuation of a buyer or a seller for a unit of the good of a certain quality reflects the discounted sum of flow utility from owning it forever\(^7\).

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\(^3\)Somewhat different outcomes may emerge in a sequential bargaining framework; see Hörner and Vieille, 2009.

\(^4\)The analysis and the outcome in Janssen and Roy (2002) are closely related to those in a model of dynamic auctions by Vincent (1990). The problems highlighted in this note have some interesting implications for dynamic auctions.

\(^5\)As explained in Janssen and Roy (2002), one can easily introduce depreciation of the kind where the quality next period is a constant fraction of the quality today; it is essentially equivalent to a change in the effective discount factor.

\(^6\)Trade history is observed so that resale, if it occurs, takes place in a different market.

\(^7\)An alternative interpretation for the seller’s valuation is that each seller has the ability (or the resource) to produce one unit of the good at a cost equal to its valuation but she must choose the time period in which to do so.
The total mass of sellers is 1. Each seller is endowed with one unit of the good. The quality of the good is either high \((\theta_H)\) or low \((\theta_L)\). We assume that \(0 < \theta_L < \theta_H\). A fraction \(\alpha \in (0, 1)\) of sellers own high quality goods. Each seller knows the quality of her own good, but the quality is not observed by buyers prior to purchase. A seller’s valuation of the good she is endowed with is assumed to be identical to its quality. Thus, the discounted net surplus (evaluated in the initial time period) of a seller who owns a unit of quality \(s\) and sells it at price \(p_t\) in period \(t\) is

\[
\left\{ \left( \sum_{i=1}^{t-1} \delta^{i-1} \right) \left( (1 - \delta) \theta_s + \delta^{t-1} p_t \right) \right\}, \text{for } t \geq 2
\]

\[p_1, \text{for } t = 1\]  

(1)

The total mass of buyers is denoted by \(\mu > 1\). Buyers have unit demand and are risk neutral. There are two types of buyers: high valuation (type \(H\)) and low valuation (type \(L\)). Let (upper case) \(V^H\) and \(V^L\) denote respectively the valuations of a high and low quality good by a high valuation buyer. For a low valuation buyer, the valuation of high and low quality goods are denoted by (lower case) \(v^H\) and \(v^L\) respectively. We assume that:

\[V^H > V^L, V^H > v^L, V^H > v^H, V^L > v^L > \theta_L\]  

(2)

Thus, independent of her type, every buyer’s valuation of the high quality good exceeds that of the low quality good. Further, for each quality, high valuation buyers are willing to pay more than low valuation buyers. A high valuation buyer’s willingness to pay for either quality good exceeds the sellers’ valuation of that quality. A low valuation buyer’s willingness to pay for the low quality good exceeds the sellers’ valuation of low quality, but this need not necessarily be true for high quality. Further, we assume that:

\[V^H - \theta_H > V^L - \theta_L\]  

(3)

i.e., a high valuation buyer creates greater social surplus when she buys a high quality good (rather than buying a low quality good); this implies that (subject to feasibility) it is socially optimal to allocate a high quality good to a high valuation buyer. The measure of high valuation buyers is denoted by \(\beta\). Recall that \(\alpha\) is the measure of high quality sellers.

We will assume that:

\[\alpha \leq \beta < 1\]  

(4)

i.e., there are enough of high valuation buyers to buy all high quality goods, but trading all goods requires that some low valuation buyers also buy.
Buyers do not observe the quality of goods offered for sale. However, they anticipate (correctly, in equilibrium) the proportion of high and low quality goods offered for sale in every period. For a high valuation buyer who anticipates that fraction $\pi_t \in [0, 1]$ of goods offered for sale in period $t$ is of high quality, the discounted net expected surplus from buying in period $t$ at price $p_t$ is given by:

$$\delta^{t-1}[p_t - \{\pi_tV^H + (1 - \pi_t)V^L\}]$$

(5)

The expected net discounted surplus of a low valuation buyer is similar. Not buying at all yields a net surplus of zero for either type of buyer.

It is easy to check that under full information, all goods are traded in the initial period, and (using (3) and (4)) all high quality goods are bought by high valuation buyers.\(^8\) This is also the first best allocation.

In order to ensure that there is a "lemons problem" in the one-period version of the model we assume:

$$\alpha V^H + (1 - \alpha)V^L < \theta_H$$

(6)

This implies that in a static market, only low quality goods are traded.

All of the assumptions outlined above are assumed to hold throughout the paper. We now look at the dynamic equilibrium outcomes under incomplete information.

**Definition 1** A dynamic equilibrium is defined by a sequence of prices $p_t \geq 0$, anticipated proportions $\pi_t \in [0, 1]$ of high quality among goods offered for sale in each period $t$, $t = 1, 2, 3...$, a purchase period for each buyer (may be infinite, which implies "never buy") and a selling period for each seller (may be infinite, which means "never sell") such that: (i) Buyers and sellers maximize their discounted expected net surplus; (ii) The market clears every period; (iii) if $t > 1$ and the total measure of low quality goods traded in periods prior to period $t$ is $(1 - \alpha)$ i.e., all low quality goods have been traded in the past, then $\pi_t = 1$ (iv) If strictly positive measure of trade occurs in any period, then the actual proportion of high and low quality goods traded in that period equals the anticipated proportions for that period.

Condition (iii) is a mild restriction on demand in periods of no trade; if (almost) all low quality goods have been sold in the past, buyers should expect quality to be

\(^8\)For example, in period 1, all low quality goods can be sold at price $v^L$ ($> \theta_L$) and all high quality goods at price $[V^H - (V^L - v^L)]$ ($> \theta_H$). H-type buyers are indifferent between buying high and low quality goods and all H-type buyers buy. L-type buyers are indifferent between buying low quality goods and not buying at all.
high with probability one. The assumptions outlined in this section are maintained in the rest of the paper.

The following properties of any dynamic equilibrium are easy to check:

R.1: In any dynamic equilibrium, a low quality seller never sells later than a high quality seller.

R.2: There can be at most one period in which both high and low quality goods are traded.

R.3: The first period in which high quality goods are sold must be strictly greater than 1.

R.4: In any dynamic equilibrium, all low quality goods are traded.

R.5: In every period $t$, the equilibrium price must satisfy

$$p_t \geq v_L$$

(7)

R.6: There can be at most one period in which only low quality goods are traded, and at most one period in which only high quality goods are traded.

R.1 follows from the fact that if a high quality seller sells in period $\tau$, then

$$0 \leq [p_\tau - \theta_H] \geq \delta^{k-1}[p_{\tau+k} - \theta_H], \forall k \geq 1$$

so that

$$0 < [p_\tau - \theta_L] > \delta^{k-1}[p_{\tau+k} - \theta_L], \forall k \geq 1$$

which implies that all low quality sellers sell in some period $t \leq \tau$. R.2 follows immediately from R.1. To see R.3, observe that if high quality sellers find it optimal to sell in period 1, then all low quality sellers would strictly prefer to sell in period 1, and this leads to a contradiction (using (6)) as even the high valuation buyers’ willingness to pay for the expected quality sold would fall short of the high quality seller’s reservation price. R.4 follows from the fact that if some low quality goods are never traded, then $p_t \leq \theta_L, \forall t \geq 1$, but since $V^L > v^L > \theta_L$ all buyers would strictly prefer to buy in some period, a contradiction (as $\mu > 1$, there must be excess demand in some period). R.5 holds as (under restriction (iii) of a dynamic equilibrium path), the willingness to pay of buyers is at least as large as $v_L$ in every time period, and there are more buyers than sellers. Finally, to see R.6, note that if there are two periods $t, t'$ in which only low quality goods are traded and, without loss of generality, $t < t'$, then for a low quality seller to be indifferent between both periods we need $p_t < p_{t'}$; however, in that case a buyer buying in period $t'$ would strictly gain by buying in period $t$ instead. The same argument holds for high quality.

Janssen and Roy (2002) show that when all buyers are homogenous and the distribution of quality is continuous on an interval, then all goods are traded in finite
time. It is easy to check that even with a discrete distribution of quality as in this paper, this result continues to hold (though for high $\delta$, trading necessarily involves intermediate periods of no trade in order to prevent waiting by low quality sellers).

**Proposition 2** *(Janssen and Roy, 2002).* Suppose that all buyers are homogenous and in particular,

$$V^H = v^H = \bar{v} > \theta_H, V^L = v^L = \underline{v} > \theta_L, \bar{v} > \underline{v}. $$

Then, there exists a dynamic equilibrium where all goods are traded in finite time. Further, the property that all goods are traded in finite time holds in every dynamic equilibrium.

**Proof.** (Sketch) If $V - \theta_L \geq \delta [\bar{V} - \theta_L]$, then all low (high) quality goods are traded in period 1 (period 2) at price $V$ ($\bar{V}$). If $V - \theta_L < \delta [\bar{V} - \theta_L]$, let $\tau \geq 1$ be the smallest positive integer such that $V - \theta_L > \delta^\tau [\bar{V} - \theta_L]$. Then, for $\epsilon > 0$ sufficiently small, there is an equilibrium where $(1 - \alpha - \epsilon)$ low quality goods are sold in period 1 at price $\underline{v}$, while $\epsilon$ low quality goods and $\alpha$ high quality goods are sold in period $1 + \tau$ at a price equal to the buyers’ expected valuation in that period. No trade occurs in periods $2, \ldots, \tau$ (set price $= \bar{V}$ in those periods; this does not violate (iii) of the definition of equilibrium as there are unsold low quality goods in the market). The argument that all goods must be traded in finite time in every dynamic equilibrium is as outlined formally in Janssen and Roy (2002). ■

### 3 Trading Problems

In this section, we show that in our model with heterogeneity of buyers, it is quite possible that dynamic trading leads to the same outcome as in a static market: only low quality goods are traded and the high quality goods are never sold.

**Proposition 3** Suppose that

$$v^H < \theta_H $$
$$\alpha < \beta < 1 - \alpha.$$  

Then no high quality good is traded in any period if either,

$$\delta < \frac{V^L - v^L}{V^H - \theta_H},$$  

or,

$$\frac{V^L - \theta_L}{\theta_H - \theta_L} < \frac{V^L - v^L}{V^H - \theta_H}. $$
Proof. (8) implies that L-type buyers’ valuation of the high quality good lies below a seller’s valuation for that quality and therefore, only H-type buyers can buy high quality goods. Using (9), though there more H-type buyers than high quality goods, we need L-type buyers in order to trade all low quality goods. Since all low quality goods are sold in every dynamic equilibrium, this implies that L-type buyers must necessarily buy. Suppose that contrary to the proposition, \( \tau \) is the first period in which a strictly positive measure of the high quality good is traded. As some high quality sellers find it optimal to sell in that period,

\[
p_{\tau} \geq \theta_H > \theta_L
\]

and using result R.1 and R.4 in Section 2, all low quality goods must be traded and in fact, must be sold on or before date \( \tau \). Since there are not enough H-type buyers to buy all low quality goods, a strictly positive measure of the low quality goods must be sold to L-type buyers. Let \( \gamma \) be the measure of low quality goods that are sold in period \( \tau \). Let \( V_{\tau}^e \) denote the expected valuation of a H-type buyer who buys in period \( \tau \). Then,

\[
p_{\tau} \leq V_{\tau}^e \leq \frac{\alpha}{\alpha + \gamma} V^H + \frac{\gamma}{\alpha + \gamma} V^L
\]

and using (6) & (12):

\[
\alpha V^H + (1 - \alpha) V^L \leq p_{\tau} \leq \frac{\alpha}{\alpha + \gamma} V^H + \frac{\gamma}{\alpha + \gamma} V^L
\]

so that

\[
\gamma < 1 - \alpha
\]

i.e., a strictly positive measure of low quality goods must have been traded in some period \( t < \tau \) (this also means that \( \tau > 1 \)). From R.6, we have that there can be at most one period prior to \( \tau \) in which low quality goods are traded i.e., \( t \) is the only period before \( \tau \) in which low quality goods are sold.

We now claim that some of the low quality goods sold in period \( t \) must have been bought by L-type buyers. To see this claim, suppose not. Then, all low quality goods traded in period \( t \) are bought by H-type buyers; since high quality goods can only be bought by H-type buyers, it follows that all goods sold prior to period \( \tau \) are bought by H-type buyers. Further, only H-type buyers buy in period \( \tau \) since \( p_{\tau} \geq \theta_H > v^H \). This implies that, in particular, all low quality goods (whose total measure is \( 1 - \alpha \)) are bought by H-type buyers (whose total measure is \( \beta \)). Using (9), we obtain a contradiction. This establishes our claim.

Observe that since L-type buyers buy in period \( t \), \( p_t \leq v_H \) and using (8), we have \( p_t < \theta_H \) which means that no high quality good is traded in period \( t \). Thus, L-type...
buyers buying in period \( t \) are willing to pay at most \( v_L \) in period \( t \); using R.5, we have that
\[
p_t = v^L 
\] (15)

An H-type buyer who buys in period \( \tau \) could also have bought a low quality good in period \( t \) and therefore:
\[
V^L - p_t \leq \delta^{\tau-t}[V^L - p^L] \leq \delta^{\tau-t}[V^H - p^H] 
\] (16)

Using (12) and (15), (16) implies:
\[
V^L - v^L \leq \delta^{\tau-t}[V^H - \theta^H] 
\] (17)

On the other hand, since some low quality sellers sell in period \( t \) instead of waiting to sell in period \( \tau \):
\[
p_t - \theta_L \geq \delta^{\tau-t}[p^L - \theta_L] 
\]

and using (12) and (15), we have:
\[
v^L - \theta_L \geq \delta^{\tau-t}[\theta^H - \theta_L] 
\] (18)

Combining (17) and (18), we obtain:
\[
\frac{V^L - v^L}{V^H - \theta^H} \leq \delta^{\tau-t} \leq \frac{v^L - \theta_L}{\theta^H - \theta_L}. 
\] (19)

which leads to a contradiction if either (10) or (11) holds.

Proposition 3 provides sufficient conditions under which there is no dynamic equilibrium in which any high quality good is sold even though the first best and the full information market outcomes are characterized by all goods being traded; intertemporal sorting and multiple trading opportunities fail to generate higher volume of trade than in the static model. Condition (8) in Proposition 3 requires that the reservation price of a high quality seller is lower than the reservation price of a low valuation buyer for the high quality good so that high quality goods can only be bought by high valuation buyers. Condition (9) requires that while there are enough high valuation buyers to buy all high quality goods, there are not enough of such buyers to buy all low quality goods so that in order to trade all low quality goods, low valuation buyers need to buy. Proposition 3 indicates that as long as these two assumptions are satisfied, no high quality good can be traded if agents are sufficiently impatient i.e., the discount factor is below a threshold \( \frac{V^L - v^L}{V^H - \theta^H} \). Note that under assumptions (2) and (3) this threshold is strictly less than 1. Proposition 3 also
indicates that even when the discount factor is above this threshold, no high quality good can be traded in equilibrium if restriction (11) is satisfied; note that (11) imposes restrictions on the valuations of buyers and sellers. All of the assumptions of our model as well as conditions (8), (9) and (11) are satisfied if, for example,

$$\theta_L = 2.1, \theta_H = 3, v^L = 2.2, V^L = 2.5, v^H = 2.5, V^H = 3.5, \alpha = 0.4, \beta = 0.5.$$ 

in which case even with arbitrarily mild discounting, high quality goods cannot be traded in equilibrium.

## 4 Inefficiency in Dynamic Allocation

Recall that in the first best allocation (full information market outcome), all high quality goods are bought by high valuation buyers. We now show that under certain conditions, the dynamic equilibrium allocation is necessarily one where some high valuation buyers buy low quality goods.

**Proposition 4** Suppose that the following hold:

$$\delta < \frac{V^L - \theta_L}{V^H - \theta_L}$$  

or

$$\delta > \frac{V^L - v^L}{v^H - \theta_L}. \quad \text{(C.2)}$$

Then either high quality goods are not traded at all, or some high quality goods are bought by high valuation buyers.

**Proof.** From R.4, we know that all low quality goods are sold in equilibrium. Suppose that contrary to the proposition, there exists a dynamic equilibrium where all high quality goods are traded and bought by H-type buyers. In such an equilibrium, all goods are sold. Let T be the first period in which high quality goods are sold. Then, from R.3, R.4 and R.6, we have that $T > 1$, some low quality goods must be sold before period $T$, and only high quality goods are sold after period $T$. As high quality goods are sold in every period $\tau \geq T$ in which trade occurs, under the supposition that all high quality goods are bought by H-type buyers with certainty, it must be true that only H-type buyers buy (if at all) in periods $\tau \geq T$ (if a positive measure of L-type buyers buy in periods $\tau \geq T$, then some L-type buyers would end up with high quality goods). Since all goods are sold and the total measure of high valuation buyers $\beta < 1$, some L-type buyers must buy, and as they do not buy in
any period \( \tau \geq T \), there exists \( t < T \) in which a positive measure of L-type buyers buy (at which time only low quality goods are sold). Since low valuation buyers buy in such a period \( t \) knowing that the quality sold is low for sure, (using (7)) it follows that

\[ p_t = v^L. \] (20)

There are two possibilities: (i) Both high and low quality goods are sold in period \( T \) (\( 0 < \pi_T < 1 \)); (ii) only high quality goods are sold in period \( T \) (\( \pi_T = 1 \)). Consider case (i). Low quality sellers are indifferent between selling in periods \( t \) and \( T \):

\[ p_t - \theta_L = \delta^{T-t}[p_T - \theta_L] \]

so that using (20), we have

\[ \delta^{T-t}p_T = v^L - (1 - \delta^{T-t})\theta_L \] (21)

On the other hand, H-type buyers buy in period \( T \) so that:

\[
V^L - p_t \leq \delta^{T-t}\{\pi TV^H + (1 - \pi_T)V^L\} - p_T < \delta^{T-t}[V^H - p_T]
\]

so that using (20), we have

\[ \delta^{T-t}p_T < \delta^{T-t}V^H - (V^L - v^L) \] (22)

From (21) & (22) we have:

\[ \delta^{T-t} > \frac{V^L - \theta_L}{V^H - \theta_L} \]

which, implies that

\[ \delta > \frac{V^L - \theta_L}{V^H - \theta_L} \] (23)

which contradicts (C.1). Next, consider case (ii). In this case, there is no time period in which both qualities are sold. Therefore, using R.6, all low quality goods are sold in some period \( t < T \), and all high quality goods are sold in period \( T \). Further, if all low quality goods are sold in period \( t \), then using part (iii) of the definition of a dynamic equilibrium, all buyers expect high quality with probability one in period \( t+1 \) which means that there is no trade in period \( t+1 \) only if \( p_{t+1} \geq V^H \) but then, all high quality sellers would strictly prefer to sell in period \( t+1 \) rather than wait for a later period because the highest price they can ever get in any period with trading is \( V^H \). Therefore, \( T = t+1 \). Since low quality sellers prefer to sell in period \( t \):

\[ p_t - \theta_L \geq \delta[p_{t+1} - \theta_L] \]
and using (20) we have
\[ \delta p_{t+1} \leq v^L - (1 - \delta)\theta_L \]  
(24)

Since L-type buyers buy in period \( t \)
\[ 0 = v^L - p_t \geq \delta[v^H - p_{t+1}] \]
so that
\[ p_{t+1} \geq v^H \]  
(25)

From (24) and (25):
\[ \delta \leq \frac{v^L - \theta_L}{v^H - \theta_L} \]  
(26)

H-type buyers buy in period \( t + 1 \) and therefore,: 
\[ V^L - p_t \leq \delta[V^H - p_{t+1}] \]
so that
\[ \delta p_{t+1} \leq \delta V^H - (V^L - v^L) \]  
(27)

From (25) and (27)
\[ \delta \geq \frac{V^L - v^L}{V^H - v^H} \]  
(28)

Both (26) and (28) must hold simultaneously so that
\[ \frac{V^L - v^L}{V^H - v^H} \leq \delta \leq \frac{v^L - \theta_L}{v^H - \theta_L} \]

and this violates (C.2), a contradiction. ■

Proposition 4 provides conditions under which there is no equilibrium where all high goods are eventually allocated to high valuation buyers. This may either occur because some or all of the high quality goods are not traded at all (as discussed in the previous section) or because a portion of such goods are bought by low valuation buyers. Note that if \( v^H - v^L < V^H - V^L \), the conditions (C.1) and (C.2) of Proposition 4 necessarily hold for small \( \delta \). Also note that unlike the conditions imposed in Proposition 3, the conditions in Proposition 4 allow for the possibility that the low valuation buyers’ willingness to pay for the high quality good exceeds the reservation price of high quality sellers so that a low value of \( \delta \) may be perfectly consistent with all goods being traded over time.

Our next proposition outlines a condition under which all goods must necessarily be traded over time, but all high quality goods are allocated to low valuation buyers. 11
Thus, while the opportunity for repeated trading and the use of waiting as a sorting mechanism may allow the volume of goods traded over time in a dynamic model to be as high as in the full information outcome, the allocation of goods across types of consumers may be quite distorted and this is an important source of inefficiency.

**Proposition 5** Suppose that $v^H > \theta_H$ and, further:

$$\delta \leq \min \left\{ \frac{V^L - v^L}{V^H - v^H}, \frac{v^L - \theta_L}{v^H - \theta_L} \right\} \quad (C.3)$$

Then, there exists a dynamic equilibrium in which all goods are traded and all high quality goods are bought by low valuation buyers.

**Proof.** We construct an equilibrium where all low quality goods are sold in period 1 and all high quality goods in period 2. All H-type buyers buy in period 1 while L-type buyers buy in periods 1 and 2. The prices are as follows:

$$p_1 = v^L, p_2 = v^H \quad (29)$$

At these prices, L-type buyers are indifferent between buying in period 1, buying in period 2 and not buying at all. A high quality seller would earn negative surplus if it sells in period 1 ($6$ implies that $v^L < V^L < \theta_H$) and strictly positive surplus in period 2 (since $v^H > \theta_H$). A low quality seller optimally sells in period 1 if:

$$p_1 - \theta_L \geq \delta[p_2 - \theta_L]$$

i.e.,

$$(1 - \delta)\theta_L \leq p_1 - \delta p_2$$

and using (29) this holds if:

$$\delta \leq \frac{v^L - \theta_L}{v^H - \theta_L}$$

which follows from (C.3). Finally, H-type buyers find it optimal to buy in period 1 if:

$$V^L - p_1 \geq \delta[V^H - p_2]$$

and using (29) this is equivalent to:

$$\delta \leq \frac{V^L - v^L}{V^H - v^H}$$

which also follows from (C.3).
5 Conclusion

In contrast to the textbook model of static market for lemons, the dynamic nature of durable goods markets and in particular, the repeated opportunity to trade and the possibility of waiting to trade have been viewed as important reasons why one should expect fairly brisk trading in such markets with even the best quality goods being traded eventually. This note indicates that when buyers are heterogenous and trading higher qualities requires higher valuation buyers to wait, the dynamic market mechanism may yield no better outcome than the static market in terms of range of qualities traded. Further, even if all goods are traded over time, the allocation of goods across buyers may be suboptimal. This adds to the inefficiency caused by waiting to trade in such markets and constitutes a new dimension of the lemons problem in dynamic markets. The results in this paper are not driven by the assumption of two quality types which is made purely for clarity of exposition (see, Roy 2011. for similar results with continuum of quality types). The results highlight the importance of allowing for significant heterogeneity among uninformed buyers in various models of dynamic trading of durable goods such as decentralized markets with frictions, as well as auctions and bargaining. They also indicate that in the presence of buyer heterogeneity, relying on dynamic interaction within a single market may not be enough to resolve the lemons problem, and therefore there is all the more reason to focus on other mechanisms (such as effective and interlinked primary and secondary markets for distinct vintages as outlined by Hendel, Lizzeri and Siniscalchi, 2004).

References


