Equilibrium Incentives to Acquire Precise Information in Delegated Portfolio Management

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Abstract

This paper investigates how a linear contract offered to a portfolio manager affects her incentives to acquire precise information. I show that increasing the manager’s portfolio share increases her demand for precise information. This result contrasts with the existing irrelevance results where the manager’s portfolio share does not affect her precision choice. The irrelevance result relies on the manager facing a constant asset price, regardless of her demand. In a noisy rational expectations framework, increasing the manager’s share decreases her demand and results in a less informative asset price. Thus, the manager gathers more precise information when offered a larger fraction of portfolio returns.

Key words: ?????????

A growing percentage of financial investment decisions are made by professional portfolio managers. Institutional ownership has increased from 6.5% in 1965 to 45% in 1991 (Allen and Gorton, 1993). The size of institutional ownership has peaked at 59.9% by the end of 1997 (Goldman and Sleazak, 2003). An important justification for the employment of professional portfolio managers is their ability to acquire and interpret information related to security returns. There is almost a consensus to describe a portfolio manager’s role by her access to superior investment information. The manager gathers information on security returns and uses this information on behalf of the investor to select a portfolio.

This paper examines how the compensation scheme of a portfolio manager affects her incentives to acquire precise investment information. The standard compensation schemes used in the money management industry, the fulcrum fees and fraction of funds fees are linear schedules. These rules compensate the portfolio manager by allocating her a share of final portfolio returns. Consistent with this actual practice, I focus on linear compensation contracts and investigate their effectiveness in motivating the manager to gather precise information. My goal is to determine whether compensating the portfolio manager with a higher share of portfolio returns induces her to gather more precise information.

I consider a portfolio manager acting on behalf of an investor. The manager’s compensation is linear in the return of the final portfolio value she generates. Prior to her portfolio choice, the manager expends costly effort and observes an information signal on the asset return. The precision of the signal depends on the manager’s effort. Moral hazard is present in the model because the effort, the precision of the signal, and the signal
realization are not observable to the investor. I find that linear contracts induce the portfolio manager to collect precise information when the market participants have rational expectations about her trades: increasing her share from the final portfolio value does give the manager better incentives for information production.

This result contrasts with some theoretical results that find that the manager’s portfolio share does not affect her precision choice at all. Although they have a different focus, Stoughton (1993) and Admati and Pfleiderer (1997) also address the problem of motivating the manager to collect precise investment information. Both papers obtain an “irrelevance result” that suggests that the manager’s portfolio share has no effect at all on her demand for precise information. My analysis illustrates that the irrelevance result relies on the portfolio manager undoing the incentive effect of the contract by her portfolio choice without affecting the asset price she faces. If her share from the final portfolio value is doubled, then the manager’s optimal demand for the asset is exactly as half as before. Assuming that the asset price she faces is constant, this response makes the stochastic part of the manager’s wealth distribution independent from her compensation. Thus, the manager’s demand for precise information does not depend on her compensation.

However, in an equilibrium setting where the asset price depends on the informed manager’s trades, the irrelevance argument does not go through. When the manager reduces her demand for the asset in response to a higher portfolio share, the resulting asset price becomes less informative (noisier). Due to this effect on the equilibrium asset price, the manager’s wealth distribution is not independent of her share of total portfolio value. Anticipating a less informative asset price, and thus a more stochastic wealth distribution, the manager demands more precise information.

Related Literature: Most papers in the literature recognize the information production function of the portfolio managers, but do not focus on the manager’s information acquisition incentives. Grinblatt and Titman (1989) and Carpenter (2000) examine the risk-taking incentives of portfolio managers in theoretical models. Davanzo and Nesbit (1987) and Grinold and Rudd (1987) find empirical evidence for increased risk-taking induced by incentive fees. Goldman and Slezak (2003) analyze the informational efficiency of stock prices in a setting where the portfolio managers’ tenure may be shorter than the time it takes for their private information to be public. Biais and Germain (2002) study the optimal contracts when a better-informed fund manager has interests that conflict with those of her clients on the investment strategy of the fund, since she also trades on her own account. Huberman and Kandel (1993) consider signaling models in the money management context, where managers use portfolio selections to signal their abilities. Das and Sundaram (2002) provide a comparison between the fulcrum fees for which the portfolio manager’s fee is symmetric around a chosen index, and option-like incentive fees which include a bonus if the portfolio return exceeds the benchmark. However, none of these papers exclusively study the manager’s information acquisition decision and the effect of the manager’s compensation on that decision. The contribution of this paper is to address how the manager’s compensation contract affects both her portfolio choice and information acquisition decision in an equilibrium setting.

The plan of the paper is organized as follows. Section 1 lays out the model. Section 2 analyzes the portfolio problem of the manager and the corresponding trading equilibrium. Section 3 characterizes the manager’s information acquisition decision and contains our
main result. Section 4 concludes. The appendix includes the proofs that are not presented in the text.

1. The model

Consider a portfolio manager who makes investment decisions on behalf of an investor. The investor gives the manager an initial wealth \( w_0 \) to invest in a portfolio. The portfolio decision involves investing in two assets, a risky asset that pays a stochastic payoff \( \tilde{v} \) and a riskless asset that pays a certain return \( r \), which is normalized to one. The distribution of the stochastic return \( \tilde{v} \) is \( N(0, 1) \). Both parties are strictly risk-averse with exponential utility functions

\[
U_A(\tilde{W}_A) = -\exp\{-a\tilde{W}_A\} \quad \text{and} \quad U_B(\tilde{W}_B) = -\exp\{-b\tilde{W}_B\},
\]

where \( \tilde{W}_A(\tilde{W}_B) \) is the final wealth of the portfolio manager (the investor), \( a > 0 \) and \( b > 0 \) are the respective risk-aversion coefficients.

The portfolio manager has access to some information technology. Prior to the portfolio decision, she can learn the value of a signal \( s \), which is correlated with the payoff of the risky asset. I assume

\[
\tilde{v} = \bar{s} + \tilde{e},
\]

where \( \bar{s} \) and \( \tilde{e} \) have a multivariate normal distribution:

\[
E[\tilde{e}] = 0, \quad E[\bar{s}\tilde{e}] = 0 \quad \text{and} \quad \text{Var}[\tilde{e}] = h^{-1} > 0.
\]

Since \( \bar{s} \) and \( \tilde{e} \) are uncorrelated, (3) implies that the posterior distribution of \( \tilde{v} \) conditional on \( s \) is given by \( (\tilde{v} \mid s) \sim N(s, h^{-1}) \). In what follows, I denote the precision of the manager’s information as \( h \). Observing a precise information signal has a private cost for the portfolio manager. An information signal with precision \( h \) costs \( c(h) \) where \( c(\cdot) \) is a standard twice differentiable strictly convex cost function. The precision choice of the manager is not observable to the investor. This assumption implies that the investor cannot directly specify a certain precision choice to the portfolio manager and compensate the manager as a direct function of that precision.

1.1. Manager’s compensation

I restrict attention to linear compensation schemes. The portfolio manager is assumed to act under a compensation contract \((F, \phi)\), where \( \phi \) is the fraction of final portfolio value \( \tilde{W} \) paid to the manager and \( F \) is a fixed fee, paid regardless of the portfolio value. This linearity assumption is consistent with the actual practice in the money management industry. The so-called fulcrum fees and fraction of funds fees widely used to compensate portfolio managers are linear and symmetric around a pre-specified benchmark (Lakonishok et al., 1992; Golec, 1992). When the precision and portfolio choices of the manager are contractible, Wilson (1968) shows that the linear rules are optimal, since they
provide optimal risk sharing. As noted, my objective in this paper is to investigate the information acquisition incentives of a manager acting under such a linear scheme. In that sense, the analysis is not an optimal contracting framework. Instead, I assume that the investor hires the manager, delegates the portfolio decision and compensates her with a linear scheme. Moreover, I implicitly assume that the investor has chosen the compensation scheme \( (F, \phi) \) such that the portfolio manager’s participation constraint \( E[U_A(\tilde{W}_A)] \geq \overline{u} \) is satisfied.

If the manager acquires \( D \) units of the risky asset at a price \( p \) and invests the rest of the initial wealth in the riskless asset, the final portfolio value \( \tilde{W} \) is given by

\[
\tilde{W} = D(\tilde{v} - p) + w_0. 
\]

The contract \( (F, \phi) \) implies that \( \tilde{W}_A \) and \( \tilde{W}_B \) are described as

\[
\tilde{W}_A = F + \phi \tilde{W} - c(h), \\
\tilde{W}_B = (1 - \phi) \tilde{W} - F. 
\]

Why does the investor hire the portfolio manager in the first place? Note that in the current framework, the safe asset pays \( r = 1 \), but without any further information the expected payoff of the risky asset is zero. Therefore, in the absence of any further information a risk averse investor would not invest at all in the risky asset by herself. Because of her access to the information signal, the portfolio manager creates an opportunity to invest in the risky asset whenever \( E[\tilde{v} | \tilde{s}] > r \), i.e., whenever there is a positive risk premium. By hiring the portfolio manager, the investor gains access to information to be used on her behalf.

1.2. Trading

Unlike the previous literature on portfolio management, I do not assume that the asset price \( p \) is constant. I use an equilibrium approach and apply the standard noisy rational expectations framework as the equilibrium concept in the asset market. Other than the portfolio manager, there are two types of traders in the asset market, an uninformed trader who also maximizes an exponential utility function with a coefficient of risk aversion \( u > 0 \), and noise traders who demand a random amount \( \tilde{z} \sim N(0, \sigma^2) \). The noise trade \( \tilde{z} \) is independent from the manager’s signal \( \tilde{s} \). The following market clearing condition determines the equilibrium in the asset market:

\[
D(p, s) + D_u(p) + z = 0, 
\]

where \( D_u \) is the uninformed demand for the risky asset.

For the reader’s convenience, the timeline in figure 1 summarizes the sequence of events.
2. Manager’s portfolio choice and the trading equilibrium

I now solve the model backwards and first characterize the manager’s portfolio decision and the corresponding trading equilibrium. My goal is to illustrate the effect of the manager’s compensation scheme on her portfolio choice and on the resulting equilibrium asset price.

Conditional on the signal s and asset price p, the manager chooses the position in the risky asset that maximizes her expected utility. Formally, the manager’s portfolio decision D is such that

$$D \in \arg \max E[U_A(\tilde{W}_A)|s,p].$$  \hspace{1cm} (8)

Given the normality assumptions and the exponential utility function, one can write the maximization problem in (8) in the mean-variance form

$$D \in \arg \max E[\tilde{W}_A|s,p] - \frac{a}{2} \text{Var}[\tilde{W}_A|s,p],$$  \hspace{1cm} (9)

with

$$E[\tilde{W}_A|s,p] = \phi D(s-p) + w_0 + F - c(h),$$  \hspace{1cm} (10)

$$\text{Var}[\tilde{W}_A|s,p] = \frac{(\phi D)^2}{h}.$$  \hspace{1cm} (10)

The solution to this problem yields the following optimal demand for the risky asset:

$$D^*(s,p) = \frac{h(s-p)}{a\phi}.$$  \hspace{1cm} (11)

Increasing the portfolio manager’s share \(\phi\) decreases her demand for the risky asset. When \(\phi\) increases, the manager’s compensation becomes more stochastic. Since she is risk averse, the manager responds to this by decreasing the position in the risky asset. This optimal demand has an interesting implication: conditional on a price \(p\) and a signal \(s\), the stochastic portion \(\phi \tilde{W}^*\) of the manager’s compensation in (5) is independent from \(\phi\). It is easy to verify that
\[
\phi W^*(s,p) = \phi D^*(\bar{v} - p) = \frac{h(s - p)(\bar{v} - p)}{a}.
\]  

(12)

In other words, for a given price and signal, the manager’s response to an increase in \(\phi\) is to decrease \(D^*\) proportionally, so that the stochastic part of her wealth is independent of \(\phi\). This argument, however, is a partial equilibrium one. By fixing \(p\), it implicitly assumes that the manager faces the same asset price regardless of her demand for the risky asset. When the asset price depends on the informed manager’s demand, then in equilibrium, the manager’s compensation \(\phi\) will affect both her demand and the equilibrium asset price she faces. Thus, the manager’s wealth distribution will not be independent of \(\phi\).

Using the portfolio manager’s demand and the market clearing condition (7), I characterize the equilibrium in the asset market and relate it to the manager’s compensation scheme.

**Proposition 1.** There is a rational expectations equilibrium in which the equilibrium asset price is linear in the signal realization and the noise trade, \(p(s,z) = \beta s + \gamma z\). The equilibrium coefficients are given by:

\[
\beta = \frac{(1 + \phi)h(h-1) + ha^2\phi^2\sigma_z^2}{(1 + \phi)h(h-1) + (h + \phi)a^2\phi^2\sigma_z^2} \quad \text{and} \quad \frac{\gamma}{\beta} = \frac{a\phi}{h}.
\]

(13)

**Proof.** See the appendix.

This explicit solution allows me to identify the effect of the portfolio manager’s compensation on the equilibrium asset price. Equation (13) shows that the equilibrium coefficient \(\beta\) is decreasing in the portfolio manager’s share \(\phi\), and \(\gamma\) is increasing in \(\phi\). In other words, the equilibrium asset price becomes less informative as \(\phi\) increases.

The intuition is as follows: increasing the manager’s share \(\phi\) lowers her optimal demand \(D^*\) as shown in (11). Thus, the demand of the noise traders becomes relatively more important in determining the equilibrium asset price and the resulting asset price \(p\) is less informative. To see this formally, note that given \(p(s,z) = \beta s + \gamma z\) and \(\bar{v} = \bar{s} + \bar{\epsilon}\), the noise in the equilibrium asset price is given by

\[
\text{Var}[\bar{v}|p] = \left[ 1 - \frac{(1-h^{-1})^2}{(1-h^{-1}) + (\frac{\gamma}{\beta})^2 \sigma_z^2} \right].
\]

(14)

Since \(\gamma/\beta = a\phi/h\), it follows that,

**Proposition 2.** Equilibrium asset price becomes less informative as the portfolio manager’s share \(\phi\) increases.

The link between the informativeness of the asset price and the portfolio manager’s information acquisition decision is a key aspect of the analysis. Proposition 2 continues to hold without the rational uninformed trader. In this case, the equilibrium asset price would be given by \(p(s,z) = s + (a\phi/h)z\) and Proposition 2 follows. The presence of an
uninformed trader has no effect on the informativeness of the equilibrium asset price and thus on the portfolio manager’s equilibrium demand for precise information.1

3. Manager’s information acquisition

I now relate the trading equilibrium to the manager’s information acquisition decision. Consider the manager’s optimal expected utility conditional on the signal (but not on the asset price). One can derive this conditional expected utility by substituting the manager’s optimal demand \( D^* \) back into her objective function. Doing so yields

\[
E[U^*(\tilde{W}_A|s)] = -E \left[ \exp \left\{ -\frac{h(s - \tilde{\rho}(\cdot))^2}{2} \right\} \right] \times \exp\{-a(\phi w_0 + F - c(h))\}. \tag{15}
\]

Using the properties of the chi-square distribution, I evaluate the expectation in (15) to obtain

\[
E[U^*(\tilde{W}_A|s)] = -\frac{1}{\sqrt{1 + h \text{Var}[\tilde{\rho}|s]}} \times \exp\{-a(\phi w_0 + F - c(h))\}. \tag{16}
\]

This last expression illustrates the link between the informativeness of the asset price and the manager’s demand for precise information. The conditional utility of the portfolio manager from a precision \( h \) is increasing in \( \text{Var}[\tilde{\rho}|s] \), the noise in the asset price.

To see the implication of a perfectly informative asset price on the manager’s information acquisition decision, suppose that \( \sigma^2 \) equals zero. Without any noise in the asset price due to the randomness of the liquidity trade, (14) implies that \( \text{Var}[\tilde{\rho}|p] = h^{-1} = \text{Var}[\tilde{\nu}|s] \). In other words, the equilibrium asset price becomes equally informative as the signal. In this case, \( \text{Var}[\tilde{\rho}|s] = 0 \) and thus the utility gains from private information disappears. As the noise in the asset price increases, the value of observing a signal with precision \( h \) increases. I state this observation in the following proposition.

**Proposition 3.** The portfolio manager’s ex ante expected utility conditional on a signal with precision \( h \) is decreasing in the informativeness of the equilibrium asset price.

The mechanism through which the manager’s compensation \( \phi \) affects her incentives to acquire precise information is as follows: The informativeness of the asset price is decreasing in the portfolio manager’s share \( \phi \). Furthermore, the portfolio manager benefits more from a given precision the less informative is the equilibrium asset price. Therefore, a portfolio manager with a higher share \( \phi \) should demand more precise information.

I conclude the analysis by deriving the manager’s optimal precision choice and verify this intuition. Formally, the portfolio manager’s precision choice \( h \) is such that

\[
h \in \arg \max E[U^*_A(\tilde{W}_A)]. \tag{17}
\]

1 I am grateful to an anonymous referee for this insightful observation.
Taking expectations over $s$ in (16), one can obtain the optimal unconditional expected utility $E[U_A(\tilde{W}_A)]$ of the portfolio manager and state her problem of precision choice as

$$h \in \arg \max - \frac{1}{\sqrt{1 + h\sigma^2(\phi, h)}} \times \exp\{ - a(\phi w_0 + F - c(h)) \}, \quad (18)$$

where

$$\sigma^2(\phi, h) \equiv \text{Var}(\tilde{s} - \bar{p}) = (1 - \beta)^2 \sigma_s^2 + \gamma^2 \sigma_{\tilde{s}}^2. \quad (19)$$

I can now illustrate the partial equilibrium nature of the irrelevance result. Suppose that the asset price $p$ is a constant. In other words, the portfolio manager faces the same asset price regardless of her demand. This setup corresponds to the framework in Stoughton (1993) and Admati and Pfleiderer (1997). In this case,

$$\sigma^2 = \text{Var}(\tilde{s}) = \sigma_s^2 = 1 - h^{-1} \quad (20)$$

and the portfolio manager’s problem of precision choice in (18) reduces to

$$h \in \arg \max - \frac{1}{\sqrt{h}} \times \exp\{ - a(\phi w_0 + F - c(h)) \}. \quad (21)$$

Therefore, when $p$ is assumed to be constant the portfolio manager’s demand for precise information becomes independent of her compensation $\phi$. I state this irrelevance result by emphasizing its partial equilibrium nature.

**Proposition 4.** If the portfolio manager faces a constant asset price regardless of her demand for the asset, then her share $\phi$ of the final portfolio value has no effect on her precision choice.

The intuition for Proposition 4 follows from the earlier discussion: If one assumes that $p$ is constant, then the stochastic part of the manager’s wealth in (5) becomes independent from $\phi$. In other words, the portfolio manager undoes the incentive effect of a higher portfolio share by adjusting her demand in the asset (without affecting the asset price), so that $\tilde{W}_A$ is independent of $\phi$. Thus, increasing $\phi$ has no effect on the manager’s demand for precise information.

However, the irrelevance result does not hold when $p$ is endogenously determined, as described in Proposition 1.

**Proposition 5.** (i) When the equilibrium asset price faced by the portfolio manager is not constant but depends on her demand, the manager’s precision choice $h$ solves

$$\frac{1}{2a} \left[ \frac{\partial \ln(1 + h\sigma^2(\phi, h))}{\partial h} \right] = c'(h). \quad (22)$$

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2 Admati and Pfleiderer (1997) derive their irrelevance result with fulcrum fees: The manager receives a base fee plus a fraction of the difference between the portfolio’s return and the return on a prespecified benchmark portfolio. Stoughton (1993) considers fraction of funds fees where the manager receives a fixed fee plus a fraction of portfolio returns.
(ii) The portfolio manager’s precision choice is increasing in her share \( \phi \) of the final portfolio value.

Proof: See the appendix.

The intuition on why the equilibrium precision \( h \) is increasing in the manager’s share \( \phi \) is worth emphasizing. When responding to a higher \( \phi \), the manager decreases her demand \( D \) proportionally. This response, in turn, increases the demand of the noise traders relative to the demand of the informed portfolio manager. Hence, the equilibrium asset price becomes less informative and the manager’s share \( \phi \) affects her wealth distribution through its impact on the informativeness of the asset price (Proposition 2). A less informative asset price increases the ex ante value of private information (Proposition 3). Anticipating a less informative asset price, the portfolio manager responds to a compensation scheme with a higher share \( \phi \) by actually gathering more precise information.

I illustrate this result with a numerical example. Let \( a = 4 \), \( \sigma^2 = 10 \) and \( c(h) = h^2/2 \). Solving the precision choice numerically for values of \( \phi \in [0, 1] \) shows that increasing the manager’s share \( \phi \) improves her precision choice (figure 2).

4. Conclusion

This paper analyzes the effectiveness of linear compensation rules in motivating the portfolio managers to acquire precise investment information. Understanding the incentive properties of linear rules is important for several reasons. First, the fraction of funds and fulcrum fees commonly used in the money management industry are linear

![Equilibrium precision](image)

**Figure 2.** A numerical example: Let \( a = 4 \), \( \sigma^2 = 10 \) and \( c(h) = h^2/2 \). The figure plots the equilibrium precision choice of the manager as a function of her share \( \phi \) of final portfolio value.
schemes. Second, the precision of the information is qualitative in nature, which implies it is difficult, if not impossible, to specify a precision choice as part of a contract. This aspect of information acquisition decision raises an agency issue of whether the manager has incentives to gather precise information. Since acquiring information on security returns is one of the major roles performed by portfolio managers, the way information acquisition depends on the manager’s compensation rule is important from a practical point of view. Finally, the literature provides an irrelevance result that suggests that the portfolio manager’s demand for precise information does not depend at all on her compensation.

By analyzing the portfolio manager’s information acquisition decision in an equilibrium context, the paper makes the following points:

(i) The manager’s contract affects both her portfolio choice and her information acquisition decision.
(ii) The irrelevance result is a partial equilibrium conclusion that relies on the implicit assumption that the asset price is constant, regardless of the manager’s demand in the asset.
(iii) When the asset price depends on the manager’s demand, then increasing her share of the final portfolio does motivate her to acquire more precise information. A practical implication of the analysis is that compensating the manager with the fulcrum fee structure commonly used in the industry, not only affects the manager’s portfolio decisions, but it also increases the manager’s incentives to acquire more precise information about these decisions.

Appendix

Proof of Proposition 1. Given the market clearing condition (7) and optimal portfolio demand $D^*$ in (11), I need to compute the uninformed demand $D_u$ to solve for the equilibrium $p$. CARA preferences and normality assumptions imply

$$D_u = \frac{E[\tilde{v} | p^*] - p}{u \Var[\tilde{v} | p^*]},$$

(A.1)

where $p^*$ is the uninformed conjecture, which is correct in equilibrium. Given the conjecture $p^*(s, z) = \beta s + \gamma z$, one can define $\tilde{\theta}(s, z) \equiv p^*/\beta$. Conditioning on $\theta$ is statistically equivalent to conditioning on the conjecture $p^*$. Therefore, I can express the signal extraction problem of the uninformed by conditioning on $\theta$ and obtain

$$E[\tilde{v} | \theta] = \theta \text{ and } (\Var[\tilde{v} | \theta])^{-1} \equiv h'' = [1 - t(1 - h^{-1})]^{-1},$$

(A.2)

where

$$t \equiv \frac{h - 1}{h - 1 + h(\gamma/\beta)^2 \sigma_z^2}.$$
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Using these expressions, I rewrite the market clearing condition (7) as

$$\frac{h(s - p)}{a\phi} + \frac{h^\prime(t_0 - p)}{u} + z = 0. \quad (A.4)$$

Substituting $\theta \equiv s + (\gamma/\beta)z$, solving for $p$, and equating it to the conjecture yields the equilibrium coefficients in Proposition 1.

**Proof of Proposition 5.** First, I derive the portfolio manager’s optimal unconditional ex ante expected utility in (18). Conditional on $s$ and $p$,

$$E[U^*(\tilde{W}_A|s, p)] = -\exp\left\{-a\left[\frac{h(s - p)^2}{2a} + \phi w_0 + F - c(h)\right]^{\frac{1}{2}}\right\}. \quad (A.5)$$

Since $\bar{p}(s, z) = \beta \bar{s} + \gamma \bar{z}$ and $\tilde{s} \sim N(0, \sigma_s^2)$ with $\sigma_s^2 \equiv 1 - h^{-1}$, it follows that

$$(\bar{s} - \bar{p}) \sim N(0, \sigma^2(\phi, h)), \quad (A.6)$$

where

$$\sigma^2(\phi, h) \equiv \text{Var}[\tilde{s} - \bar{p}] = (1 - \beta)^2 \sigma_s^2 + \gamma^2 \sigma_z^2.$$ 

Now define $\Gamma \equiv (\bar{s} - \bar{p})^2/\sigma^2(\phi, h)$. $\Gamma$ has a chi-square distribution with one degree of freedom. Therefore,

$$E[U^*(\tilde{W}_A)] = E\left[\exp\left\{-\frac{h|\sigma^2(\phi, h)\Gamma|}{2}\right\}\right] \times \exp\{-a(\phi w_0 + F - c(h))\}. \quad (A.7)$$

For a chi-square distribution with one degree of freedom, the moment generating function is given by $E[\exp(t\Gamma)] = 1/\sqrt{1 - 2t}$ (Hogg and Craig, 1978). Letting $t = -h|\sigma^2(\phi, h)|/2$ then allows me to evaluate the above expectation and obtain the objective function in (18). Equation (22) which describes the portfolio manager’s optimal precision $h$ follows from taking a monotone transformation of (18) and differentiating it with respect to $h$. The comparative statics result in part (ii) follows from totally differentiating the equilibrium condition in (22) with respect to $h$ and $\phi$, and noting that $\sigma^2(\phi, h)$ is strictly increasing in $\phi$.

**Acknowledgments**

I am grateful to the editor, Haluk Unal, and two anonymous referees for their helpful comments. Ms. Sandra S. Moore provided valuable editorial assistance. I would also like to thank Charles A. Wilson, Alberto Bisin, Douglas Gale, Boyan Jovanovic, Levent Kockesen, Richard Barnett, and seminar participants at Midwest Finance Association 2003 meetings in St. Louis. The usual disclaimer applies.
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