Financial Innovations and Managerial Incentive Contracting

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Abstract

The top executives’ demand for financial instruments that enable them to reduce the risk exposure in their compensation has increased drastically in the last decade. We analyze the implications of the manager’s access to financial innovations and hedging ability for incentive contracting. We show that when the manager’s access to hedging is restricted to a fixed number of trading rounds with risk neutral third parties, then the hedging ability does not affect the equilibrium effort at all. If the manager has access to potentially infinite trading rounds with risk neutral third parties, however, she hedges completely and no incentives can be sustained. Therefore, restricting the manager’s access to a fixed number of rounds is crucial for sustaining incentives.

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1 Introduction

The median compensation of S&P 500 CEOs increased by approximately 150 percent from 1992 to 1998, with stock and option based compensation providing the largest share of gains. The 1990s witnessed a dramatic increase in the top executives receiving compensation in the form of company shares or options on the company shares.\(^1\) During that decade, the top executives' demand for financial instruments that can reduce the risk exposure in their compensation also increased. An article in The Economist reports that the use of derivatives to eliminate managerial exposure to firm risk has become a business of hundred millions of dollars.\(^2\) According to Bettis et al. (2001), there has been a huge increase in the development, sophistication and use of strategies that enable corporate insiders to hedge their stock ownership positions in their firms.\(^3\)

The agency theory on managerial compensation contracts justifies the use of performance based reward schemes by the need to align the shareholder-manager incentives.\(^4\) Linking the manager’s wealth to firm performance by using stock based compensation gives the manager the right incentives to maximize shareholder value. However, it seems that the managers are able to trade in the financial markets and unilaterally adjust the risk exposure in their compensation. By engaging in hedging transactions with third parties, the managers can potentially reduce the sensitivity of their wealth to firm

\(^1\) According to an article in Financial Times by Chris Giles on Feb 5, 2002, between 1992 and 1998, the base salary of an average CEO increased by 29 percent, the average bonus increased by 99 percent and average value of options granted rose by 335 percent. Also see the New York Times Article ‘The Outrage Constraint’ by Paul Krugman (August 23, 2002).


\(^3\) Bettis et al. (2001) empirically examine the use of two hedging instruments: zero-cost collars (the simultaneous purchase of a put option funded by the proceeds received from the sale of a call option on the company’s stock) and equity swaps (exchange of the returns on the firm’s stock for the cash flows on another asset such as a risk-free security).

performance, and undo the intended incentive effect of stock based compensation. Alarmed by the possibility that high ranking corporate insiders might use hedging instruments to undo their incentive contracts, some practitioners recommend firms to severely restrict or preclude the use of these instruments (Schizer (2000)).

A somewhat controversial issue is the permissive attitude of firms toward managers in determining the extent and the timing of their hedging transactions. Bebchuk et al. (2002) and Bebchuk and Fried (2003) argue that firms take surprisingly few steps to prevent or regulate the financial market access of their executives: ‘Executives are left free to hedge away the equity exposure and typically are permitted to choose the amount and timing of unwinding (Bebchuk et al. (2002), page 76)’. They further assert that the optimal contracting approach is quite at odds with the managers’ apparent freedom to hedge away the risk in their compensation: if the manager can potentially undo the incentive effect of the contract, why would the optimal contract allow the manager to trade with a third party and alter the pay-performance sensitivity freely? An editorial in *The Economist* agrees with this assertion:5 ‘Hedging is almost never prohibited...Logically, boards should restrict and control the sale of bosses’ shares.’

In this paper, we formally analyze the implications of the managers’ access to financial innovations and their resulting hedging ability for incentive contracting. To this end, we extend the standard principal-agent setting and consider the possibility that the manager can trade with third parties and unilaterally alter her risk exposure once her compensation contract is set by the firm. In particular, we follow the empirically documented hedging practice by Bolster et al. (1996) and Bettis et al. (2001) and focus on a commonly observed type of managerial trading which is known as an *equity swap*. In an equity swap transaction, the manager promises the third party to pay the return on her company’s stock, in exchange for the cash flows on another asset such as a risk-free security.

5’Taken for a ride’, The Economist, July 11, 2002.
Our analysis addresses the following questions: (1) Does the manager completely undo the incentive contract when she can trade with third parties or does she face a trade-off when choosing her optimal hedge size? (2) How does the optimal pay-performance sensitivity compare to the case when the manager has no access to trading? (3) How does the equilibrium effort choice of the manager compare when she can and cannot trade? Does allowing the manager to hedge the compensation contract undermine incentives completely?

We first consider a setting where the manager is restricted to trade a fixed number of rounds. We find that

(1) The manager does not undo her compensation contract completely. This result follows since the terms of the manager’s hedging transaction depends on the incentives she retains, i.e., what she does not hedge.

(2) The equilibrium pay-performance sensitivity is higher compared to the case when manager can not hedge. As long as the manager does not hedge completely, her access to hedging transactions implies that the principal has to worry less about insuring the manager (because she does it herself) and more about giving the manager incentives. Accordingly, the equilibrium pay-performance sensitivity is strictly higher with hedging. This result may explain why the managers’ greater access to financial innovations has been accompanied by even higher pay-performance sensitivities in executive compensation in the last decade.

(3) The manager’s hedge affects the principal’s problem of optimal effort inducement only when the third parties trading with the manager are risk averse. If the third parties are risk neutral, then the equilibrium effort is the same with or without hedging. This result follows because with risk neutral third parties, the manager does not pay a risk premium in her hedging transaction and hence fully internalizes the risk-incentive trade-off of the principal. Therefore, the cost of inducing the agent to a certain effort level and hence the surplus as a function of the induced effort is the same with or without hedging.
We then allow the manager to engage in potentially infinite trading rounds and show that

(4) Restricting the number of rounds manager can trade is crucial for sustaining incentives. With potentially infinite rounds, equilibrium effort is always adversely affected. The only allocation that survives the possibility of further trading is the one that achieves optimal risk sharing between the manager and the third parties. In particular, with potentially infinite rounds, incentives are completely undermined with third party risk neutrality: the manager hedges completely and the lowest possible effort is implemented in equilibrium.

It is probably a good approximation to assume risk neutrality for the third parties as investment banks trading with the manager. Provided third party risk neutrality, our results indicate that if the manager is restricted to trade only fixed known number of rounds, then hedging ability does not affect the equilibrium effort implemented. With potentially infinite trading opportunities with risk neutral parties, no incentives can be sustained. A key consideration for sustaining incentives is that the contracts available to the manager in the hedge market must be exclusive, so that their price depends on the quantity traded. This is the case if the manager is restricted to trade a fixed number of rounds.\(^6\) To deny the manager the ability to engage in infinite side trades, the compensation committee can implement a restriction that the manager can trade, say twice every year with advance permission from the committee. It is interesting to note that restricting the manager’s trades in this way is one of the suggestions made by Bebchuck et al. (2003) in their critique of the principal-agent model of executive compensation: "One could adopt a variety of restrictions on the timing of sales without hindering the executive’s ability to satisfy legitimate liquidity and diversification needs. For example, one could require that sales are carried out gradually over a specified period or one could require the executive to receive advance permission from the compensation committee before trading."

\(^6\) I would like to thank to an anonymous referee and Michel Poitevin for this observation.
In general, two considerations are important for the equilibrium implications of manager’s trades on the effort implemented. The first one is whether the manager hedges completely or not. If the optimal hedge is such that all risk is hedged, then incentive contracting fails completely and the lowest possible effort is implemented. This is the case with potentially infinite trades with risk neutral third parties. The only allocation that survives further trading possibilities is the one driven only by risk sharing (but not incentives) considerations and the manager hedges completely. On the other hand, if the manager is restricted to trade a fixed number of times, retaining some incentives becomes important. In this case, the hedging transaction is exclusive and it trades off between benefits and cost of diversification. Hedging is costly since (i) the manager pays a risk premium to third parties if they are risk averse (ii) even if they are risk neutral, third parties price the hedging transaction according to their rational conjecture of the manager’s subsequent effort choice. Due to this trade-off, the manager does not hedge completely.

The principal anticipates the manager’s trades with third parties and designs the contract accordingly. If the manager’s trades are exclusive and hence she retains some incentives, the second consideration that comes into play is how the manager’s hedge affects the principal’s effort inducement problem. A negative externality arises if the third parties are risk averse and the manager pays a risk premium in the hedging transaction. We show that this negative externality disappears either (naturally) when hedging is not possible at all or when the third parties are risk neutral. As a function of the effort implemented, the surplus is the same with or without hedging if the third parties are risk-neutral and hedging is exclusive. Therefore, in this case the equilibrium effort implemented is not affected by hedging. If third parties are risk averse and require a risk premium, effort inducement is more costly compared to the case with no hedging and equilibrium effort is hence lower.
*Related Literature:* Three recent papers also address the managers’ ability to alter the risk in their compensation. A common theme in these papers is that the optimal compensation contract should take into account the risk averse manager’s ex post incentives to diversify away the systematic risk (Garvey and Milbourn (2003) and Jin (2002)) or substitute between systematic and firm-specific risk factors (Acharya and Bisin (2002)). In contrast to our analysis, these papers preclude the manager’s ability and incentives to diversify the firm level risk by assumption. Jin (2002) shows that by taking a short position in the market portfolio, the manager can completely diversify away all the systematic (market) risk in her compensation. Garvey and Milbourn (2003) also study the case when the manager can only hedge the systematic risk. Their main conclusion is that if it is costly for the manager to diversify the systematic risk on her own, then her compensation should depend on the firm’s market adjusted performance, a practice commonly called *relative performance evaluation*. They also find empirical support for this prediction. In contrast to these papers, we allow the manager to diversify away the firm-specific risk. In practice, many of the managerial hedging instruments are focused on the firm specific risk rather than the systematic risk. This is especially true in case of equity swaps and zero-cost collars, the two most common hedging instruments manager use (see Bettis et al. (2001), page 348).

The most closely related paper to this paper is Bettis et al. (2001) which also addresses the managers’ access to financial innovations. Bettis et al. (2001), however, does not provide a model to formally analyze the impact of managerial hedging on incentive contracting. The main conclusion of their empirical analysis is that the executives use financial innovations like zero cost collars and equity swaps (i) primarily for risk reduction purposes and (ii) as a substitute for insider selling when a large sale would be most likely to attract attention. Our paper contributes to this relatively new literature by formally analyzing the determinants of the manager’s demand for a financial innovation and how the access to that financial innovation changes the nature of the trade-off in incentive contracting.
The paper is organized as follows. The next section describes the model and formally lays out the contract problem between the principal and the manager. Section 3 presents the analysis and contains the results. Section 4 discusses the implications of the analysis and concludes. Proofs not presented in the text are contained in the Appendix A. In Appendix B, a more general framework illustrates the robustness of our results.

## 2 The Model

We extend the standard principal-agent setting to incorporate for the possibility that the agent can trade with third parties after her compensation contract is set. The basic ingredients of the model are as follows:

**Technology and Preferences:** An agent (the manager) runs a firm owned by a principal (the shareholder). The principal is risk neutral and maximizes the final firm value net of the manager’s compensation. The manager has exponential preferences with a constant absolute risk aversion coefficient $a > 0$. The final value of the firm, $\tilde{X}$, is determined by the following stochastic technology $\tilde{X} = e + \tilde{\epsilon}$ where $e$ is the costly and unobservable effort expended by the manager and $\tilde{\epsilon}$ is the stochastic component over which the manager has no control. For tractability, we assume that the manager’s cost of effort is given by $c(e) = ke^2/2$ with $k > 0$, a constant. Furthermore, we employ the standard normality assumption on the distribution of $\tilde{\epsilon}$ and assume that $\tilde{\epsilon} \sim N(0, \Sigma)$.

**Manager’s Compensation:** Drawing on the optimality results in Holmstrom and Milgrom (1987), we restrict attention to linear compensation contracts. In particular, the manager’s compensation contract is described by a pair $(F, s)$ where $F$ is a fixed payment and $s$ is the manager’s share of the final firm value. Accordingly, the manager’s compensation is given by

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7 See Appendix B for a more general framework.

8 The optimality of the linear sharing rule in Holmstrom and Milgrom (1987) depends critically on the constant absolute risk aversion utility function for the manager. With more general preferences, a linear contract might not be optimal. Jin (2002) points out that in practice the sharing rule is often close to linear, because the convexity induced by the manager’s stock options is negligible to the first order.
In what follows, we refer to $s$ as the pay-performance sensitivity of the manager's compensation scheme.

**Trading with Third Parties:** We depart from the standard principal-agent setting and allow the manager to trade with third parties before she makes her effort choice. This approach has been common in the literature. All the existing papers that address the implications of the manager’s ability to trade with third parties on agency contracts (Garvey (1993), Jin (2002), Garvey and Milbourn (2003), and Bisin and Guaitoli (2004)) exclusively study settings where such trading takes place before the effort decision.\(^9\)

Legal constraints and regulations that restrict the managers’ ability to trade in the primary stock market as insiders are commonplace. However, our motivation is different from a standard insider trading model where an informed manager trades in the stock of her own company. Instead, following the empirically documented hedging practice by Bolster et al (1995) and Bettis et al (2001), we allow for a commonly observed type of managerial trading which is known as an equity (or diversification) swap.\(^{10}\)

In an equity swap transaction, the manager agrees to pay the third party (usually an investment bank) the return from the firm’s stock, and the third party pays the manager the return from an investment such as a fixed-income security (see Braddock (1997) and Gastineau (1991) for a complete discussion of how swaps are structured). We consider a transaction where the manager pays the third party $\alpha X$ in exchange for a fixed payment $G$. Notice that by engaging in a swap transaction, the manager can unilaterally alter the link between her compensation and shareholder wealth and undo her incentive contract: she can potentially set $\alpha = s$ and eliminate all the risk in her compensation.

\(^9\)Fudenberg and Tirole (1990) considers an agency setting where the initial contract is renegotiated after the effort choice. As we discuss in Section 3.6, their results would apply to our setting if we had focused on hedging after the effort decision.

\(^{10}\)Bettis et al (2001) note that unlike open market trades (the standard insider trading practice), the managerial trading in equity swaps and other hedging instruments (like zero cost collars) are less likely to attract market scrutiny. They point out that in general, the services that provide insider trading data to the financial markets do not make available the information needed to identify the instrument in the hedging transactions.
There is a continuum of third parties that the manager can trade. They act competitively in forming their demands for the swap transaction, as we specify shortly. The third parties have exponential preferences with a constant absolute risk aversion coefficient \( b > 0 \). Furthermore, all third parties have rational expectations about the manager’s subsequent effort choice given her eventual holdings of the company’s share and they perfectly observe the manager’s initial contract. We analyze two different trading environments. First, we impose the restriction that the manager can engage in only a fixed number of trading rounds. In this setting, we characterize the manager’s optimal hedge, and effort decisions and the pay-performance sensitivity of the compensation contract. Then in Section 3.5 we remove this restriction and allow the manager to engage in potentially infinite trading rounds.

Before we formally lay out the contract problem, we summarize the sequence of events in our model with the timeline below.

\[
\begin{align*}
&t=0 & t=1 & t=2 & t=3 \\
\text{Principal offers} & \text{Manager trades} & \text{Manager chooses effort} & \text{Firm value is realized.} \\
\text{the manager a linear} & \text{with third parties} & \text{effort} & \\
\text{compensation} & \text{Scheme (F,s)} & \text{choices} & \\
\text{Scheme (F,s)}
\end{align*}
\]

\[2.1 \quad \text{The Contract Problem}\]

The principal optimally sets the compensation rule \((F, s)\) taking into account the subsequent trading \((\alpha)\) and effort \((e)\) choices of the manager. Given a compensation rule \((F, s)\) and an action pair \((\alpha, e)\), the manager’s wealth distribution is given by

\[
\tilde{W}_m(s, F, \alpha, e) = [F + s\tilde{X}] + [G - \alpha\tilde{X}] - c(e). \tag{1}
\]

With the normality assumption on the distribution of \(\tilde{\varepsilon}\) and CARA preferences, the manager’s expected utility can be written in the mean-variance
form. The complete formulation of the contract problem is as follows:

\[ \max_{(F,s)} E \left[ (1-s)\bar{X} - F \right] \quad \text{subject to} \]

\[ E \left[ \tilde{W}_m(\alpha^*, e^*) \right] - (a/2) \text{Var} \left[ \tilde{W}_m(\alpha^*, e^*) \right] \geq 0 \]  

(2)

\[ \alpha^* \in \arg \max E \left[ \tilde{W}_m(s, F, \alpha, e^*) \right] - (a/2) \text{Var} \left[ \tilde{W}_m(s, F, \alpha, e^*) \right] \]  

(3)

\[ e^* \in \arg \max E \left[ \tilde{W}_m(s, F, \alpha^*, e) \right] - (a/2) \text{Var} \left[ \tilde{W}_m(s, F, \alpha^*, e) \right] \]  

(4)

(2) is the manager’s participation constraint where we normalize the manager’s reservation payoff to zero. (3) and (4) describe the manager’s optimal trading and effort choices, respectively.

3 Analysis

3.1 Effort Choice

Suppose the manager receives a compensation contract \((F, s)\) and then she trades \(e\bar{X}\) in exchange for \(G\). The mean and the variance of her wealth distribution \(\tilde{W}_m\) are given by \(E \left[ \tilde{W}_m \right] = F + (s - \alpha)e + G - c(e)\) and \(\text{Var} \left[ \tilde{W}_m \right] = (s - \alpha)^2 \Sigma\). Accordingly, the manager’s optimal effort choice that solves (4) is given by \(e^*(s, \alpha) = (s - \alpha)/k\). If the manager sets \(\alpha = s\) in the trading stage and eliminates all the uncertainty in her compensation, then she does not expend any effort and sets \(e^* = 0\). Note, however, that the payment \(G\) she will receive in exchange for \(\alpha\bar{X}\) depends on her subsequent effort choice. We analyze the manager’s choice of \(\alpha\) next.

3.2 Equilibrium with Fixed Number of Rounds

We first analyze the model under the assumption that the manager is restricted to trade either only once or a fixed known number of rounds. The third parties all observe the manager’s initial contract and they have rational expectations about the manager’s subsequent effort incentives. The manager chooses her hedge size \(\alpha\) strategically taking into account the effect
of this choice on the payment schedule $G(s, \alpha) \equiv \alpha p(s, \alpha)$, where $p(s, \alpha)$ is the price in a single swap transaction.

A key feature of the analysis with fixed number of rounds is that the trading opportunity available to the manager is exclusive so that the price of the transaction depends on the quantity traded. The manager’s trading partners know the total hedge size $\alpha$ and price the transaction accordingly. The market clearing payment $G(s, \alpha)$ is determined as follows: A third party chooses his demand $d^*(\alpha, p)$ for the swap such that

$$d^* \in \arg \max (d-p)E \left[ \tilde{X} \mid s, \alpha \right] - (b/2)(d-p)^2 Var \left[ \tilde{X} \mid s, \alpha \right]$$

(5)

where he takes $p$ as given. Since $E \left[ \tilde{X} \mid s, \alpha \right] = e^*(s, \alpha)$, the above problem yields

$$d^*(\alpha, p) = [e^*(s, \alpha) - p]/b \Sigma$$

(6)

In equilibrium, the total supply $\alpha$ offered by the manager must clear the demand or $\alpha = [e^*(s, \alpha) - p(s, \alpha)]/b \Sigma$. We report the market clearing fee schedule $G(s, \alpha)$ in the following lemma.

**Lemma 1** The fee schedule $G(s, \alpha)$ is given by

$$G(s, \alpha) = \alpha e^*(s, \alpha) - b \Sigma \alpha^2$$

(7)

The first term $\alpha e^*(s, \alpha)$ in the fee schedule represents the rational assessment by the third parties that once the manager hedges $\alpha$ of her holdings, she sets $E \left[ \tilde{X} \mid s, \alpha \right] = e^*(s, \alpha)$. The last term $b \Sigma \alpha^2$ represents the adjustment (or discount) for risk. Now one can derive the manager’s optimal choice of $\alpha$ that solves (3). Note that the manager’s wealth distribution $\tilde{W}_m(\alpha, e^*(\cdot))$ yields

$$E \left[ \tilde{W}_m(\alpha, e^*(s, \alpha)) \right] = se^*(s, \alpha) - b \Sigma \alpha^2 + F - c(e^*(s, \alpha))$$

$$Var \left[ \tilde{W}_m(\alpha, e^*(s, \alpha)) \right] = (s - \alpha)^2 \Sigma$$

(8)
The following first order condition describes the manager’s optimal hedge $\alpha$:

\[
\frac{a \Sigma (s - \alpha)}{\text{marginal benefit of hedging}} = -\frac{\partial e^*(s, \alpha)}{\partial \alpha} \alpha + 2b \Sigma \alpha. \tag{9}
\]

The manager faces a trade-off in her hedging decision for two reasons. One reason is the risk adjustment in the fee schedule: the term $2b \Sigma \alpha$ stands for the marginal cost of hedging due to the third party risk aversion.\(^{11}\) The second reason is that hedging decreases the manager’s subsequent incentives to expend costly effort. When the manager increases $\alpha$ marginally, the third parties correctly assess the manager’s subsequent lower effort choice and thus $G(s, \alpha)$ goes down by $-(\partial e^*(s, \alpha)/\partial \alpha)\alpha$. Our first proposition reports the manager’s optimal hedge when she is restricted to a fixed number of trading rounds.

\textbf{Proposition 2} (i) The optimal hedge is given by

\[
\alpha^*(s) = \left( \frac{a \Sigma}{a \Sigma + 2b \Sigma + 1/k} \right) s. \tag{10}
\]

(ii) The manager does not hedge completely. The manager’s hedge $\alpha^*$ is increasing in the firm level risk $\Sigma$, her risk aversion $a$, her pay-performance sensitivity $s$ and the cost of effort parameter $k$. It is decreasing in the bank’s risk aversion $b$.

Proof: See Appendix A.

Note that even if the third parties are risk neutral, the manager’s hedge is not complete. To illustrate clearly the manager’s trade-off in her hedging decision, suppose $b = 0$, and hence $G(s, \alpha) = ae^*(s, \alpha)$. The manager’s optimal hedge is then determined by the trade-off between eliminating more risk from her compensation (increasing $\alpha$) and the cost of this diversification (less incentives to exert effort and hence lower $G$). This trade-off can also

\(^{11}\)Recall that due to third party risk aversion, the manager suffers a discount $b\Sigma \alpha^2$ in the fixed payment schedule $G(s, \alpha)$
be observed from the manager’s optimal hedge ratio in (10). After setting $b = 0$, the hedge ratio becomes

$$t = a\Sigma/[a\Sigma + (1/k)]. \quad (11)$$

$a\Sigma$ measures the importance of diversification for the manager; high risk aversion ($a$) and/or high level of firm risk ($\Sigma$) increases the demand for hedging. On the other hand, $1/k$ (inverse of the cost of effort parameter) measures the extent of the effort moral hazard problem which determines the cost of diversification. When $k$ is very large, it is clear that with or without diversification, the manager’s effort will be low anyway. Therefore, maintaining subsequent effort incentives to charge a higher $G$ is less of an issue for the manager. In this case, the manager’s hedge ratio $t$ approaches to 1 and she diversifies completely. When $k$ is low, the adverse effect of diversification on the subsequent effort incentives and hence on the payment $G$ is more pronounced and the manager diversifies less.

To summarize, Proposition 2 shows that the manager does not completely undo the incentive contract when she is restricted to a fixed number of trading rounds. In this case, the manager faces a diversification-incentives trade-off in the choice the optimal hedge size. To fully characterize the impact of managerial hedging on incentive contracting, we next analyze how the principal responds to the manager’s hedging ability in setting the optimal pay-performance sensitivity.

### 3.3 Equilibrium Pay-Performance Sensitivity

The principal sets the linear contract $(F, s)$ to maximize $(1 - s)e^*(s, \alpha^*(s)) - F$ subject to the manager’s optimal hedge $\alpha^*(s)$ described in (10), the optimal effort $e^*(s, \alpha^*(s)) = (s - \alpha^*(s))/k$ and the individual rationality constraint in (2). Given $\alpha^*(s)$ and $e^*(s, \alpha^*(s))$ the manager’s individual rationality constraint becomes

$$se^*(s, \alpha^*(s)) - b\Sigma(\alpha^*(s))^2 + F - c(e^*(s, \alpha)) - (a/2)\Sigma(s - \alpha^*(s))^2 \geq 0 \quad (12)$$
In equilibrium, this constraint holds as an equality. Solving for $F$, one can state the principal’s problem as

$$s^* \in \arg \max e^*(s) - c(e^*(s)) - b\Sigma (\alpha^*(s))^2 - (a/2)\Sigma(s - \alpha^*(s))^2 \quad (13)$$

It is convenient to characterize the equilibrium pay-performance sensitivity as a function of the manager’s hedge ratio $t$. Note that given CARA preferences, the manager’s optimal hedge $\alpha^*(s)$ is linear in $s$, i.e., $\alpha^*(s) = ts$.

We first describe the optimal $s$ as a function of $t$ to illustrate the effect of manager’s trading ability on the optimal pay-performance sensitivity.

**Proposition 3** The optimal pay-performance sensitivity as a function of the manager’s hedge ratio $t$ is

$$s(t) = \frac{1}{ak\Sigma + \left(\frac{1 - 2t}{1 - t}\right)} \quad (14)$$

Proof: See Appendix A.

An immediate corollary follows by considering the case when $t \in (0, 1)$, i.e. when the manager hedges a positive fraction of her risk exposure but not all of it.

**Corollary 4** When the manager hedges a fraction $t \in (0, 1)$ of her risk exposure, the optimal pay-performance sensitivity is strictly higher compared to the case when she has no access to hedging.

We can now directly characterize the equilibrium pay-performance sensitivity by substituting for the equilibrium $t$ in (10) into (14).

**Corollary 5** Suppose the manager is restricted to trade fixed number of rounds. The equilibrium pay-performance sensitivity $s^*$ is given by

$$s^* = \frac{1}{1 + 2bk\Sigma(1 + ak\Sigma)} \quad (15)$$

If the third parties trading with the manager are risk neutral, then $s^* = 1$. 

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We again discuss the intuition for the above result for the case when the third parties are risk neutral \((b = 0)\) and hence \(s^* = 1\). Recall that the manager’s subsequent choice of the hedge size \(\alpha\) fully takes into account the risk-incentive trade-off. The optimal strategy for the principal then is to set the pay-performance sensitivity as if the manager is risk neutral. In other words, the principal completely ignores any risk sharing considerations in the choice of the contract when the manager can hedge her risk exposure. The principal only cares for the incentive problem and lets the agent deal with the risk-incentive trade-off. The manager fully internalizes this trade-off, since the fixed payment \(G = \alpha e^*(s, \alpha)\) is determined by the effort incentives she retains.

### 3.4 Equilibrium Effort with Hedging

In order to analyze the effect of manager’s hedging ability on the equilibrium effort implemented, it is convenient to rewrite the principal’s contract choice problem described in (13). Note that we have \(\alpha^*(s) = ts\) and \(e^*(s, \alpha) = s(1 - t)/k\). Hence, one can rewrite the problem in (13) with effort \(e\) being the choice variable as

\[
e^* \in \arg \max e - c(e) - (a/2)(ke)^2 - b(\Sigma k^2(\frac{t}{1 - t})^2 e^2)
\]

This last expression clearly illustrates that the manager’s access to hedging instruments affects the principal’s problem of optimal effort inducement only when the third parties trading with the manager are risk averse, i.e., when \(b > 0\). If the third parties are risk neutral, then the problem of optimal effort inducement is independent from the manager’s hedging behavior. We first derive the optimal effort as a function of manager’s hedge ratio \(t\).

**Proposition 6** The optimal effort as a function of the manager’s hedge ratio \(t\) is given by

\[
e(t) = \frac{1}{k[1 + ak\Sigma + 2kb\Sigma(\frac{t}{1 - t})^2]}
\]
Proof: Follows from maximizing (16) with respect to $e$.

The above proposition illustrates that if the third parties trading with the manager are risk averse and hence require a risk premium ($b > 0$), then the manager’s hedging ability adversely affects the equilibrium effort implemented, since for $b > 0$, $e(t)$ is decreasing in $t$. An interesting result that emerges from the analysis is that when the third parties are risk neutral ($b = 0$) or when the manager is not allowed to trade at all ($t = 0$), the equilibrium effort is the same. We state this observation in the following proposition.

**Proposition 7** Suppose the third parties trading with the manager are risk neutral and the manager is restricted to trade a fixed number of rounds. Then the manager’s hedging ability has no effect on the equilibrium effort implemented and

$$e^* = \frac{1}{k(1 + ak\Sigma)}$$

which is the same effort choice if the manager is not allowed to trade with third parties.

The intuition for the above result is as follows: Consider the contracting problem with the principal and the manager when the manager is not allowed to hedge. The surplus as a function of the optimal effort is given by

$$e - c(e) - (a/2)\Sigma(ke)^2$$

where the last term $(a/2)\Sigma(ke)^2$ is the risk premium that must be paid to the risk averse manager to induce him to an effort level $e$ by exposing her to a risk level $s = ke$. Now consider the same problem when the manager can hedge by trading with third parties with risk aversion coefficient $b > 0$. In this case, the surplus as a function of $e$ is

$$e - c(e) - (a/2)\Sigma(ke)^2 - b\Sigma(e^*)^2.$$
The additional last term \( b\Sigma(\alpha^*)^2 \) stands for the risk premium the manager has to pay to the risk averse third parties when she hedges an amount \( \alpha^* \) for which she must be compensated by the principal ex ante. Given \( \alpha^* = ts \) and \( s(1 - t) = ek \), one can translate this hedging cost to a function of effort as \( b\Sigma(\alpha^*)^2 = b\Sigma k^2(t/1 - t)^2e^2 \). This extra cost disappears either when the manager does not have to pay a risk premium to her trading partners \((b = 0)\) or when she is not allowed to hedge at all \((t = 0)\). To summarize, when the manager is restricted to a fixed number of trading rounds, the hedging ability affects equilibrium effort inducement only when it introduces the principal this extra cost due to the risk aversion of the manager’s trading partners.

It is illustrative to compare the above analysis with the one in Bisin and Guaitoli (2004). They analyze a model with two possible effort choices and two outcomes and provide conditions for low effort equilibria where the agent gets full insurance eliminating any incentive scheme completely. An important difference is that theirs is a common agency setting where all financial intermediaries offer contracts simultaneously to a single agent (multiple principals).\(^{12}\) In our setting, one principal moves first and then the agent can contract with third parties following the initial contract.\(^{13}\) Another important difference is that in our setting the price of the hedging transaction \( G(s, \alpha) \) depends on the hedge size and therefore the manager faces a diversification-retained incentives trade-off. In Appendix B, we show that our incomplete hedge result holds in a more general setting as long as the the sign of \( \partial G(s, \alpha)/\partial \alpha \) evaluated at \( s = \alpha \) is negative. Hence our result does not depend on the CARA preferences, normality assumption or the linearity of the incentive contract.\(^{14}\)

\(^{12}\)They analyze credit card and insurance markets where multiple companies make simultaneous offers to the consumer.

\(^{13}\)In that sense, our framework is more closely related to the sequential contracting setting in Bizer and Demarzo (1992).

\(^{14}\)See Appendix B for a more general setting.
3.5 Equilibrium with Potentially Infinite Trading Rounds

In this section, we analyze the equilibrium of the model when the manager is not restricted to trade a fixed number of rounds. In particular, we now consider a trading environment where after any trading round further trading is possible. When the third parties anticipate that the manager might trade further after each trade, then their demands for the swap transaction are no longer characterized by (6). Hence, the manager’s optimal trade in Proposition 2 is no longer an equilibrium.

In a trading environment with potentially infinite trading rounds, one has to look for an equilibrium allocation where the manager chooses not to trade any further, even if further trading is possible. In order to characterize such an equilibrium allocation, we adopt Admati, Pfeiderer and Zechner’s (APZ, 1994) globally stable allocation as the equilibrium concept. This concept requires that although the manager can not commit to a last round of trade, once this allocation is reached she will not trade any further.

First, we need to define a fee schedule for the swap transaction which is consistent with a globally stable allocation. Let \( \theta \) denote the manager’s holdings of company risk after she sequentially trades away \( \alpha \) starting with some initial holdings \( s \). Suppose the third parties conjecture that the final holdings of the manager will be \( \theta^* \). Then in each trading round they will be willing to trade only at

\[
\Phi(\delta, \theta^*; s) \equiv \delta[e^*(s, s - \theta^*) - \delta \Sigma(s - \theta^*)]
\]

(21)

if the manager offers to swap \( \delta \hat{X} \).\(^{15}\) Following APZ (1994), we will say that an allocation \( \theta^* \) is globally stable if and only if

(i) the third parties conjecture that the final holdings of the manager will be \( \theta^* \) and hence they will be willing to trade only at \( \Phi(\delta, \theta^*; s) \) if the manager offers to swap \( \delta \hat{X} \) in that trading round. Given that fee schedule, the manager will have no incentive to trade away from \( \theta^* \). Therefore, if one

\(^{15}\)Note that the optimal effort is given by \( e^*(s, s - \theta^*) = \theta^*/k \).
starts from a globally stable allocation $\theta^*$, the conjecture that $\theta^*$ will be the manager’s final holdings of company risk is justified.

(ii) starting from any initial holdings $s \in [0,1]$ such that $s \neq \theta^*$, if the third parties conjecture that the final holdings of the manager will be $\theta^*$ (and hence they will be willing to trade only at $\Phi(\delta, \theta^*; s)$), the manager has the incentive to trade from $s$ to $\theta^*$ at that price.

The following proposition characterizes the globally stable allocation of the trading game with potentially infinite trades.

**Proposition 8** Suppose that $a>1/k$. Then the unique globally stable equilibrium of the trading game with potentially infinite rounds is that the manager hedges

$$a^+(s) = \left(\frac{a}{a+b}\right)s. \quad (22)$$

If $a\Sigma < 1/k$, then there is no globally stable equilibrium of the trading game.

Proof: See Appendix A.

The above proposition shows that the only allocation that survives when further trading is possible is the one that achieves optimal risk sharing between the manager and the third parties. This allocation is determined only by risk sharing considerations and as in APZ (1994) and Bizer and DeMarzo (1994) it corresponds to the competitive equilibrium allocation. With potentially infinite rounds of trading, the particular effort technology does not effect the hedging behavior of the manager. Furthermore, unlike the case with fixed number of rounds, when the third parties are risk neutral, the manager completely undoes her incentive contract and hedges completely. Accordingly, when $b = 0$, the subsequent effort choice will be zero as well. We conclude this section by characterizing the equilibrium effort implemented when the manager can trade potentially infinite times.

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16Bizer and DeMarzo (1994) show that when buyers and sellers cannot commit to a single round of trade, market participants must anticipate future trading opportunities when considering whether to make or accept offers in the present. They show that the Coase conjecture holds, i.e., the monopolist loses all monopoly power when she can not commit not to reduce the price of the good when consumers are patient.
Proposition 9 Suppose that the manager can trade potentially infinite rounds and $a\Sigma > 1/k$. The equilibrium effort implemented is given by

$$e^{**} = \frac{1}{k[1 + ak\Sigma + 2k\Sigma(a^2/b)]}$$

(23)

and it is increasing in the risk aversion coefficient $b$ of the third parties trading with the manager. If the third parties are risk neutral, then $e^{**} = 0$.

Proof: Substituting $t = a/(a+b)$ as the manager’s hedge ratio in Proposition 6 yields (23). Note that if $b = 0$, we have $e^{**} = 0$.

This result shows that when considering the effect of manager’s trades on the equilibrium effort, the exclusivity of the trading opportunity available to the manager (a fixed number of rounds) is crucial. With potentially infinite trading rounds with risk neutral third parties, the only allocation that survives further trading possibilities is the one that completely insures the manager. Since the manager obtains full insurance for any incentive contract offered, the principal can not this time restore any incentives by appropriately designing the initial contract and the equilibrium effort is zero.

3.6 Hedging After The Effort Decision

Throughout the analysis, we assumed that the manager’s hedging transactions take place before she actually implements her effort. In this section, we discuss the implications of a model where the manager hedges after the effort decision: After the principal offers an incentive contract $(F, s)$, the manager first chooses $e$ and then trades with third parties who do not observe her effort. The trading between the manager and the third parties now involves an adverse selection problem with the effort chosen before trade takes place being the manager’s type.

A similar setting has been analyzed by Fudenberg and Tirole (1990) in the context of a renegotiation problem where the principal and the agent can renegotiate the initial contract once the agent chooses her effort. They
show that the only pure strategy effort choice that can be implemented is the lowest possible effort: Suppose the principal offers a contract to implement an effort choice \( e > 0 \). In equilibrium, if the principal is certain that \( e \) is implemented, then in the renegotiation stage the optimal contract would fully insure the agent with the risk neutral principal bearing all the risk. Anticipating full insurance on the equilibrium path, the agent would then choose \( e = 0 \).\(^{17}\) This argument holds in our setting as well when the third parties trading with the manager are risk neutral. Consider an equilibrium where the principal offers a contract to implement an effort level \( \hat{e} > 0 \). Suppose further that the third parties also perfectly infer this effort and hence they are willing to trade at \( G = \alpha \hat{e} \). A type \( \hat{e} \) manager’s optimal hedge \( \alpha^{**} \) then maximizes \((s - \alpha)e + F + \alpha \hat{e} - (a/2)\Sigma(s - \alpha)^2\) which implies that the manager of type \( \hat{e} \) hedges completely, i.e., \( \alpha^{**}(\hat{e}, s) = s \). But anticipating this full insurance scheme, the manager ex ante chooses the lowest effort level, and hence \( e^{**} = 0 \). Accordingly, in a setting where the manager trades with risk neutral third parties after she chooses her effort, the only equilibrium effort in pure strategies that the principal can induce is \( e^{**} = 0 \).

**Remark:** The manager’s hedging ability in our setting can be viewed as the agent renegotiating her contract with a third party other than the principal.\(^{18}\) Therefore, it is possible to relate our analysis to the renegotiation literature.\(^{19}\) We have shown that if the manager is restricted to trade a fixed number of rounds with risk neutral third parties before choosing her effort, the equilibrium effort is unaffected. This result is a reminiscent of the result in Beaudry and Poitevin (1995) who show that interim renegotiation (that takes place before action is chosen) does not constrain the allocations

\(^{17}\)Therefore, if renegotiation is possible, the equilibrium must involve mixed strategies with the principal inducing also low effort levels with high enough probability so that the contract is not renegotiated.

\(^{18}\)I am grateful to Michel Poitevin for this interpretation and also bringing the results in Fudenberg and Tirole (1990) to my attention.

attainable with full commitment (in our case no ability to hedge), but ex-post renegotiation (that takes place after the action is chosen) does. Our analysis, however, suggests that the sequential structure of the game alone is not enough to predict whether effort choice will be affected or not. The exclusivity of the hedge transaction and third party risk preferences are also important. When hedging is before the effort decision, effort choice is also affected with (i) fixed number of trading rounds (exclusivity) with risk-averse third parties or (ii) potentially infinite rounds (nonexclusivity).

4 Implications and Conclusion

In this section, we discuss some implications of our analysis and conclude. When necessary, we emphasize if the implication is also consistent without manager committing to a last trading round.

Implication 1: The use of private hedging instruments may not be necessarily detrimental to managerial incentives. However, the manager’s access to hedging must be scrutinized by the firm and restricted to fixed number of trades. We illustrated that if the manager is restricted to trade a fixed number of times, then with risk neutral third parties the access to hedging instruments does not undermine incentive contracting at all. In their empirical analysis of the implications of managerial hedging for incentive contracting, Bettis et al. (2001) address this issue as well. They examine the stock price performance subsequent to managerial hedging transactions. They do not find any statistically significant support for the hypothesis that the manager’s hedging ability undermines effort incentives. Our analysis indicates, however, that the manager’s access to hedging should be restricted to a fixed number of times. As suggested by Bebchuck et al. (2003), this can be achieved by requiring the manager to carry out trades gradually and with advance permission from the compensation committee.

Implication 2: Managers who experience large increases in the value of their equity positions demand hedging instruments more, i.e., an increase in the pay-performance sensitivity $s$ increases the hedge size $\alpha$. This predic-
tion is consistent with or without the manager committing to a last round of trade (see Propositions 2 and 8). Intuitively, it is a straightforward implication of the manager’s risk reduction motive for using the hedging instruments. Bettis et al. (2001) find that the demand for hedging instruments is positively related to whether the firm recently went public. Going public decision provides a good experiment to test the determinants of the manager’s hedging demand for two reasons: First, the insiders experience large increases in the value of their equity positions when the firm goes public. Our formal analysis predicts that this large increase would also increase the proportion of managerial wealth tied to the firm performance and hence should increase the demand for hedging. Second, insiders of firms that have recently gone public are often subject to lock-up provisions that prohibit them from immediately selling their shares (see Brav and Gompers (2003)). As we mentioned before, using equity swaps and similar hedging instruments is a better option for the insiders than directly trading in the primary stock market, since it attracts less market scrutiny. Therefore, one would expect a sharp increase in the corporate insiders’ demand for hedging instruments shortly after their firm goes public.

Implication 3: In general, the introduction of hedging instruments that enable the managers to privately reduce their risk exposure increases the equilibrium pay-performance sensitivity. As long as manager does not hedge completely (which is the case only with potentially infinite trading rounds and risk neutral third parties), the hedging ability actually increases the pay-performance sensitivity. The underlying economic reasoning for this prediction is as follows: When the manager gains access to hedging instruments to alter her risk exposure unilaterally, the principal worries less about exposing the manager to firm specific risk. Recall that in a standard setting where the manager cannot hedge, the risk sharing consideration prevents the principal from increasing pay-performance sensitivity in the first place. When the agent can hedge, the principal’s problem of designing an optimal compensation scheme to induce the agent the right incentives does not have to deal with insuring the agent (by exposing her to less risk). Since the
manager subsequently changes her risk exposure according to her risk-return preferences, the principal can increase the pay-performance sensitivity. To the best of our knowledge, this implication of our analysis has not been empirically tested.

**Implication 4:** Managers of companies with high stock price volatility demand hedging instruments more. This prediction holds when the manager is restricted to trade a fixed number of rounds and it directly follows from the manager’s hedging choice $\alpha$ derived in Proposition 2. Bettis et al. (2001) find supporting empirical evidence for this prediction. They document that purchases of equity swaps and zero cost collars by corporate insiders are followed by an increase in the volatility of stock returns of their firms.

In this paper, we attempt to provide some answers and develop insights for the implications of managerial hedging on incentive contracting. For sustaining incentives, a key consideration is that the trading opportunity available to the manager in the hedge market is exclusive so that the price of the transaction depends on the quantity traded. Commitment to a final trade achieves this exclusivity since the manager’s trading partners know the total quantity she trades and price the transaction accordingly. In this case, with third party risk neutrality the same effort is implemented. If the manager has access to potentially infinite trades with risk neutral third parties, she hedges completely and no incentives can be sustained. Therefore, the manager’s ability to hedge may not be necessarily detrimental for incentives, however restricting this access to a fixed number of trading rounds is crucial.
Appendix A: Proofs

Proof of Proposition 2: The optimal $\alpha^*(s)$ follows directly from (10) and that $\partial e^*(s, \alpha)/\partial \alpha = -1/k$. The comparative statics results on part (ii) are straightforward observations from (10).

Proof of Proposition 3: Differentiating (13) with respect to $s$, one gets

$$(1 - c) \frac{\partial e^*}{\partial s} (1 - \frac{\partial \alpha^*}{\partial s}) - a\Sigma(s - \alpha^*)(1 - \frac{\partial \alpha^*}{\partial s}) - 2b\Sigma\alpha^*\frac{\partial \alpha^*}{\partial s} = 0 \quad (24)$$

The manager’s optimal hedge $\alpha^*$ solves (9) and hence we have $a\Sigma(s - \alpha^*) = -(\partial e^*/\partial \alpha)\alpha^* + 2b\Sigma\alpha^*$. Substitute this equality above and note that $\alpha^*(s) = ts$ and $\partial e^*/\partial s = -(\partial e^*/\partial \alpha) = 1/k$. Rearranging and solving for $s$ yields (14).

Proof Proposition 8: For convenience, define

$$\Gamma(x; s) \equiv xe^*(s, s - x) - c(e^*(s, s - x)) - (a/2)x^2\Sigma. \quad (25)$$

Given the definition in the text, if an allocation $\theta^*$ is globally stable, then

$$0 \in \arg \max_{\delta} \Gamma(\theta^* + \delta; s) - \Phi(\delta, \theta^*; s). \quad (26)$$

Differentiating with respect to $\delta$ and evaluating the first order condition at $\delta = 0$, we obtain

$$(\theta^* - c'(e^*(s, \theta^*)))\frac{\partial e^*}{\partial \theta} - a\Sigma\theta^* + b\Sigma(s - \theta^*) = 0 \quad (27)$$

The optimality of the effort decision implies that $\theta^* = c'(e^*(s, \theta^*))$, hence the above expression simplifies to

$$\theta^* = \left(\frac{b}{a + b}\right)s \quad (28)$$

Finally, we need $\Gamma(x; s)$ to be concave in $x$ so that the second order condition for a maximum is satisfied, which implies that $a\Sigma > 1/k$. 

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Appendix B: A More General Setting

We now sketch a more general framework than the linear contracting, normal distribution and CARA setting analyzed in the text. In this setting, we generalize the result that if the manager can commit to a final trading round when trading with risk neutral third parties before the effort decision, then hedging ability does not affect the equilibrium effort implemented.

Let the final value of the firm $\tilde{x}$ be distributed with a probability density function $f(x,e)$ where $e \in [e_L, \infty)$ is the manager’s costly unobservable effort. We assume that the expected firm value $\int x f(x,e)dx$ is increasing in $e$. The risk neutral principal offers the manager a sharing rule $\phi(x)$ which specifies the manager’s payment if the final firm value is $\tilde{x} = x$. Upon receiving $\phi(x)$, the manager can trade with risk neutral third parties. In particular, we assume that the manager can swap a fraction $t \in [0,1]$ of her compensation in exchange for a fixed payment $G(t)$. We also assume that the manager can commit to a final trading round. Since the third parties are risk neutral, we have

$$G(t) = \int t\phi(x)f(x,e^*(t))dx$$

where $e^*(t)$ is the manager subsequent optimal effort choice after hedging a fraction $t$ of her compensation scheme. The manager’s utility is separable in wealth and effort, i.e., $u(w,e) = v(w) - c(e)$ where $w$ is manager’s final wealth, $c(e)$, the disutility from effort is convex and increasing in $e$ with $c(e_L) = 0$ and $v(.)$ is strictly concave. Accordingly, the contract problem is described as follows: The principal sets the compensation rule $\phi(x)$ to maximize $\int (x - \phi(x))f(x,e^*(t^*))dx$ subject to

$$\int v((1-t^*)\phi(x) + G(t^*))f(x,e^*)dx - c(e^*) \geq \bar{u}$$

$$t^* \in \text{arg max } \int v((1-t)\phi(x) + G(t))f(x,e^*(t))dx$$

$$e^* \in \text{arg max } \int v((1-t^*)\phi(x) + G(t^*))f(x,e)dx - c(e)$$
Proposition 10 If the fee schedule $G(t)$ satisfies $\partial G(t = 1)/\partial t < 0$, then we have $t^* \in (0, 1)$, i.e., the manager does not hedge completely.

Proof: Let $e^*(t)$ denote the solution to the manager’s effort problem in (32). Clearly $e^*(t = 1) = e_L$ and $e^*(t)$ is decreasing in $t$, i.e., if the manager hedges all of her compensation contract, she subsequently sets $e^* = e_L$. Furthermore, the more she hedges, the less incentives she has to expend effort. If the manager hedges completely, the corresponding fee she receives is given by $G(1) = \int \phi(x) f(x, e_L) dx \equiv \epsilon$. Now consider the manager’s problem of choosing the optimal hedge size in (31) and let $\Psi(t, x) \equiv v((1 - t)\phi(x) + G(t)) f(x, e^*(t))$. In order to prove the proposition, it suffices to show that

$$ \int \left( \frac{\partial \Psi(t = 1)}{\partial t} \right) dx < 0. \quad (33) $$

Differentiating $\Psi(.)$ with respect to $t$ and evaluating at $t = 1$, we obtain

$$ \frac{\partial \Psi(t = 1)}{\partial t} = \nu'(\epsilon) \left( -\phi(x) + \frac{\partial G(t = 1)}{\partial t} \right) f(x, e_L) + v(\epsilon) \frac{\partial f(x, e)}{\partial e} \frac{\partial e^*}{\partial t} < 0 \quad (34) $$

Proposition 11 The equilibrium effort implemented is the same with or without hedging, when the manager can commit to a final trading round with risk neutral third parties.

Proof: Suppose the manager is not allowed to trade and for this case an optimal contract is $\phi^*(x)$ which implements an effort level $e^*$. Then the following first order condition for effort choice must hold:

$$ \int v(\phi^*(x)) \frac{\partial f(x, e)}{\partial e} dx = c'(e^*). \quad (35) $$

Now, suppose the manager is allowed to trade and under the conditions of Proposition 10, she hedges a fraction $t^* \in (0, 1)$ of her compensation scheme.
in exchange for \(G(t^*)\). Let us define

\[ \phi^{**}(x) \equiv \frac{\phi^*(x) - G(t^*)}{1 - t^*} \]  

(36)

If the principal offers \(\phi^{**}(x)\) and the manager hedges a fraction \(t^*\) of \(\phi^{**}(x)\), her choice of effort must satisfy

\[
\int v(\phi^{**}(x)(1 - t^*) + G(t^*)) \frac{\partial f(x, e)}{\partial e} dx = \int v(\phi^*(x)) \frac{\partial f(x, e)}{\partial e} dx = c'(e) 
\]

(37)

which implies that \(\phi^{**}(x)\) induces the same effort that would be implemented without hedging. Furthermore, \(\phi^{**}(x)\) satisfies the manager’s participation constraint as an equality:

\[
\int v(\phi^{**}(x)(1 - t^*) + G(t^*)) f(x, e^*) dx - c(e^*) = \int v(\phi^*(x)) f(x, e^*) dx - c(e^*) = \bar{u} 
\]

(38)

since, by definition, \(\phi^*(x)\) is optimal without hedging and both \(\phi^{**}(x)\) and \(\phi^*(x)\) implement the same effort \(e^*\). Finally, we need to show that implementing \(e^*\) by offering \(\phi^{**}(x)\) is indeed optimal for the principal. The surplus from implementing a certain effort \(e\) is given by \(\int x f(x, e) dx - c(e)\) minus the risk premium that must be paid to the risk averse manager to implement \(e\). But note that, with or without hedging, the manager’s ex ante expected utility from implementing \(e^*\) is given by \(\int v(\phi^*(x)) f(x, e^*) dx - c(e^*)\). Therefore, the risk premium that must be paid to the risk averse manager to implement \(e^*\) is the same with or without hedging and consequently, implementing \(e^*\) by offering \(\phi^{**}(x)\) is optimal for the principal.
References


