Johny moves to a new neighborhood and must confront the bully (P2). With probability 0.8 Johny is strong (Type S) and can defeat the bully in a fight. With probability 0.2, he is Weak (Type W) and bully will win a fight. Unfortunately for P2, he can not observe Johny’s type. However, P2 can observe what Johny orders in the bar before the confrontation. From past experience, he knows that Strong Johny (Type S) prefers Beer (B) and Weak Johny (Type W) prefers Quiche (Q).
A Perfect Bayesian Equilibrium requires players

- To maximize their payoffs given their beliefs about the type of each player.
- Whenever possible, these beliefs must be consistent with the actions that players take in the game.

In this game Johny’s choice of beer or quiche sends a signal to (P2) about Johny’s type. Given this signal (beer or quiche), (P2) forms beliefs about Johny’s type and decides to fight or not.
For example, in an equilibrium in which Type S plays B and Type W plays Q (Separating Equilibrium), upon observing B, (P2) must believe that Johny is Type S with probability 1, whereas upon observing Q (P2) must believe Johny is Type W with probability 1.
In an equilibrium in which both Type S and Type W play Beer (Pooling Equilibrium), upon observing B, (P2) must believe that Johny is Type S with probability 0.8 and must believe Johny is Type W with probability 0.2.
Let $p$ denote the probability (P2) assigns to Johny being type S upon observing B. Hence $1 - p$ is the probability (P2) assigns to Johny being type W upon observing B. If (P2) fights upon observing B, he gets $(1 - p)$ whereas if (P2) retreats he gets $p$. Hence upon observing B (P2) fights if and only if

$$(1 - p) > p \Rightarrow p < \frac{1}{2}$$

and retreats if $p \geq \frac{1}{2}$. 

Let $\theta$ denote the probability (P2) assigns to $$ upon observing B. Hence $1 - \theta$ is the probability (P2) assigns to $$ upon observing B. If (P2) fights upon observing B, he gets $(1 - \theta)$ whereas if (P2) retreats he gets $\theta$. Hence upon observing B (P2) fights if and only if

$$(1 - \theta) > \theta \Rightarrow \theta < \frac{1}{2}$$

and retreats if $\theta \geq \frac{1}{2}$. 

### Saltuk Ozerturk (SMU)  
Signaling Games
Let $q$ denote the probability (P2) assigns to Johny being type S upon observing Q. Hence $1 - q$ is the probability (P2) assigns to Johny being type W upon observing Q. If (P2) Fights upon observing Q, he gets $(1 - q)$ whereas if (P2) retreats he gets $q$. Hence upon observing Q (P2) fights if and only if

$$(1 - q) > q \Rightarrow q < \frac{1}{2}$$

and retreats if $q \leq \frac{1}{2}$. 
Therefore, after observing B or Q, (P2) fights if and only if he believes Johnny is Type S with a probability less than 1/2.
Is there a **Separating PBE in which Type S plays B and Type W plays Q**? In this PBE, upon observing Q, (P2) believes that Johny is Type W with probability 1 and he fights. Upon observing B, (P2) believes that Johny is Type S with probability 1 and he retreats. But Type W will deviate from this equilibrium. If Type W plays Q he gets 1 whereas if he plays B he would get 2.

There is no **Separating PBE in which Type S plays B and Type W plays Q**.
Is there a **Separating PBE in which Type S plays Q and Type W plays B**? In this PBE, upon observing Q, (P2) believes that Johny is Type S with probability 1 and he retreats. Upon observing B, (P2) believes that Johny is Type W with probability 1 and he fights. But Type W will deviate from this equilibrium. If Type W plays B he gets 0 whereas if he plays Q he would get 3.

**There is no Separating PBE in which Type S plays Q and Type W plays B.**
Is there is a **Pooling PBE in which both Type S and Type W play B**? In this PBE, upon observing B, (P2) believes J is Type S with probability 0.8 and he retreats. Since Q is not observed we must assign off-equilibrium beliefs for the event that (P2) observes Q. Suppose if (P2) observes Q he believes J is Type S with probability $q < 1/2$ and hence he fights. Type S has no incentive to deviate. If type S plays B he gets 3, whereas if he plays Q he gets 0. Type W does not deviate either. If type W plays B he gets 2, whereas if he plays Q he gets 1. This is a PBE!
There is a Pooling PBE in which both Type S and Type W play B. In this PBE, upon observing B, (P2) believes J is Type S with probability 0.8 and he retreats. To sustain this PBE, off equilibrium beliefs of (P2) is such that if he were to observe Q (P2) assigns a probability $q < 1/2$ that J is Type S and he fights.

Why do we need (P2) to fight if he were to observe Q to sustain this PBE?
Is there a **Pooling PBE in which both Type S and Type W play Q?** In this PBE, upon observing Q, (P2) believes J is Type S with probability 0.8 and he retreats. Since B is not observed we must assign off-equilibrium beliefs for the event that (P2) observes B. Suppose if (P2) observes B he believes J is Type S with probability $p < 1/2$ and hence he fights. Type S has no incentive to deviate. If type S plays Q he gets 2, whereas if he plays B he gets 1. Type W does not deviate either. If type W plays Q he gets 3, whereas if he plays B he gets 0. This is a PBE!
There is a Pooling PBE in which both Type S and Type W play Q. In this PBE, upon observing Q, (P2) believes J is Type S with probability 0.8 and he retreats. To sustain this PBE, off equilibrium beliefs of (P2) is such that if he were to observe B (P2) assigns a probability \( p < 1/2 \) that J is Type S and he fights.

Why do we need (P2) to fight if he were to observe B to sustain this PBE?
So we have no Separating PBE!

We have two Pooling PBE

- **Pooling PBE #1**: Both Type S and Type W choose Beer and (P2) retreats after observing Beer. Off equilibrium, if he were to observe Q, (P2) assigns a probability $q < 1/2$ that J is Type S and he fights.

- **Pooling PBE #2**: Both Type S and Type W choose Quiche and (P2) retreats after observing Quiche. Off equilibrium, if he were to observe B, (P2) assigns a probability $p < 1/2$ that J is Type S and he fights.

The off equilibrium beliefs to sustain **Pooling PBE #2** make little sense. One can impose a further refinement on these beliefs to eliminate **Pooling PBE #2, but this is beyond the scope of our class :)**