Common Stock Valuation

- Common stock represents an ownership interest in a corporation, but to the typical investor it is simply a piece of paper characterized by two features:
  
  - it entitles its owner to dividends, but only if the company has retained earnings out of which the dividends can be paid, and only if the management chooses to pay dividends rather than retaining and reinvesting the cash. In short, the company is not obliged to pay dividends to its stockholders.
  
  - the stock can be sold at some future date, hopefully at a price greater than the purchase price.

- Like all financial assets, equilibrium stock prices are the present value of a stream of cash flows, which, in case of stocks, is the present value of future dividends the stock is expected to generate.

\[
\text{Value of stock} = P_0 = \frac{D_1}{1 + r_s} + \frac{D_2}{(1 + r_s)^2} + \ldots \frac{D_n}{(1 + r_s)^n} + \ldots
\]

where \(D_n\) is the expected dividend at date \(n\), and \(r_s\) is the rate of return that investors require to hold that stock.

Constant Dividend Growth Model for Stock Valuation

- This model is based on the assumption that the dividends are expected to grow at a constant rate \(g\). In this case, if the last dividend that the company paid is \(D_0\), we will have

\[
\begin{align*}
D_1 &= D_0(1 + g), \\
D_2 &= D_1(1 + g) \\
&\quad \ldots \\
D_{t+1} &= D_t(1 + g)
\end{align*}
\]
Accordingly, the current stock value will be given by

\[ P_0 = \frac{D_1}{1 + r_s} + \frac{D_2}{(1 + r_s)^2} + \ldots + \frac{D_n}{(1 + r_s)^n} + \ldots \]

\[ = \frac{D_0(1 + g)}{1 + r_s} + \frac{D_0(1 + g)^2}{(1 + r_s)^2} + \ldots + \frac{D_0(1 + g)^n}{(1 + r_s)^n} + \ldots \]

\[ = \frac{D_0(1 + g)}{1 + r_s} \left( 1 + \frac{1 + g}{1 + r_s} + \left(\frac{1 + g}{1 + r_s}\right)^2 + \ldots + \left(\frac{1 + g}{1 + r_s}\right)^n + \ldots \right) \]

which yields

\[ P_0 = \frac{D_0(1 + g)}{r_s - g} = \frac{D_1}{r_s - g} \]

- In other words, in a constant growth dividend stock model, the current stock price is given by the next expected dividend divided by \( r_s \) divided \( g \), where \( r_s \) is the required return on the stock and \( g \) is the expected constant dividend growth rate.

**Example 1:** Consider a stock that just paid a dividend of $2 per share. The investors expect the dividends to grow at a constant annual rate \( g = 8\% \). If the investor require a rate of return of \( r_s = 13\% \) for holding the company stock, what should be the current stock price?

**Answer:**

\[ P_0 = \frac{D_0(1 + g)}{r_s - g} = \frac{2(1 + 8\%)}{13\% - 8\%} = 43.2 \]

- **Capital Gains Yield and Dividend Yield in a Constant Growth Model:**

Consider the stock price valuation formula for the constant dividend growth model given by

\[ P_0 = \frac{D_1}{r_s - g} \]

This equation tells us that the stock price at date 1 is given by

\[ P_1 = \frac{D_2}{r_s - g} \]
which implies that the percent change in the stock price from date 0 to date 1 is equal to
\[
\frac{P_1 - P_0}{P_0} = \frac{\frac{D_2}{r_s - g} - \frac{D_1}{r_s - g}}{r_s - g} = \frac{D_2 - D_1}{D_1} = \frac{D_1(1 + g) - D_1}{D_1} = g
\]
In other words, in a constant dividend growth model, the stock price grows at a constant rate \(g\) equal to the dividend growth rate. Accordingly, capital gains yield for this stock is given by \(g\).

**Decomposition of the Required Return on the Stock**

Again consider the stock price valuation formula for the constant dividend growth model given by
\[
P_0 = \frac{D_1}{r_s - g}
\]
This equation can be rewritten as
\[
r_s - g = \frac{D_1}{P_0}
\]
\[
\Rightarrow r_s = \frac{D_1}{P_0} + g
\]
Therefore, the required return on the stock can be decomposed into two components
\[
r_s = \frac{D_1}{P_0} \underbrace{+ \ g}_{\text{capital gains yield}}
\]

**Example 2:** Consider a stock that just paid a dividend of $5 per share. The investors expect the dividends to grow at a constant annual rate of \(g = 5\%\). If the investor require a rate of return of \(r_s = 10\%\) for holding the company stock, what is the expected dividend yield? What is the expected capital gains yield?

**Answer:** First, find the current stock price as
\[
P_0 = \frac{D_0(1 + g)}{r_s - g} = \frac{$5(1 + 5\%)}{10\% - 5\%} = 105
\]
Now one can find,
\[
\text{Dividend Yield} = \frac{D_1}{P_0} = \frac{$5(1 + 5\%)}{105} = 5\%
\]
\[
\text{Capital Gains Yield} = g = 5\%
\]