Bonds and Their Valuation

- A bond is a long term contract under which a borrower (the issuer) agrees to make payments of interest and principal on specific dates, to the holders of the bond.

- Bonds are classified into 4 main types
  - **Treasury Bonds**: (also referred to as government bonds) are issued by the U.S. federal government. No default risk but their values are subject to interest rate risk.
  - **Corporate Bonds**: issued by corporations. Unlike Treasury bonds, the corporate bonds are subject to default risk (credit risk)- if the issuing company gets into financial trouble, it may be unable to make the promised interest and principal payments.
  - **Municipal Bonds**: are issued by the state and local governments. The interest earned from municipal bonds is exempt from federal income tax if the holder is a resident of the issuing state. Due to that tax advantage, municipal bonds carry interest rates that are considerably lower than those on corporate bonds with the same default risk.
  - **Foreign Bonds**: are issued by foreign governments or corporations. They are subject to default risk, interest rate risk and currency risk.

**Key Characteristics of Bonds**

- **Par Value**: The par value is the stated face value of the bond; for illustrative purposes we generally assume a par value of $1000. The par value generally represents the amount of money that the issuer borrows and promises to repay on the maturity date.

- **Coupon Interest Rate**: This rate determines the interest payment that the issuer promises to make at equal time intervals, typically every six months (semianual coupon payments). A bond
with semiannual coupon payments and with an **annual coupon rate** of 8% promises coupon payments of

$$1000 \left( \frac{8\%}{2} \right) = 40$$

every six months until the maturity date.

- **Maturity Date:** Bonds generally have a specified maturity date on which the par value must be repaid.

**Bond Valuation:** Like any financial asset, the value of a bond is the present value of the cash flows that the bond is expected to produce. To illustrate the basics of bond valuation, consider a 10-year bond with semiannual coupon payments. The annual coupon rate is set at 10%. Until its maturity, this bond promises the following cash flow structure:

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>$50</td>
</tr>
<tr>
<td>$1000</td>
<td>$50</td>
</tr>
</tbody>
</table>

In the above cash flow structure, we find the semiannual coupon payments as

$$\text{par value} \left( \frac{\text{annual coupon rate}}{2} \right) = 1000 \left( \frac{10\%}{2} \right) = 50$$

- To find the present value of the above cash flow structure, we need to discount the cash flows at the bonds market rate of interest $r_d$. This discount rate is also called the *yield* or the *going interest rate*. Note that $r_d$ is not the coupon interest rate. It is equal to the coupon interest rate only if the bond is selling at par. Generally, most coupon bonds are issued at par, i.e., the coupon rate is set equal to $r_d$ at the time of the issue date. Thereafter, the interest rates measured by $r_d$ will fluctuate, but the coupon rate is fixed.

- Accordingly, suppose that at the time when the bond is first issued, the ongoing annual interest rate on the issuer is $r_d = 10\%$ and hence
the annual coupon rate is also set equal to 10%. Therefore, the value of the bond at the issue date will be given by

\[
\text{Bond value at Year 0} = 50(PVIFA)_{5\%,20} + 1000(PVIF)_{5\%,20}
\]

\[
= 50(12.462) + 1000(0.377)
\]

\[
= 623 + 377 = 1000
\]

Again, since the coupon rate is set equal to the ongoing interest rate, the bond value is $1000, i.e., the bond trades at par.

- Now consider the same bond a year later, i.e., after the first two coupon payments are made. The remaining cash flows on the bond are as follows:

\[
\begin{array}{ccc}
\text{Year 1} & \text{Year 2} & \text{Year 10} \\
$50 & $50 & $1000 \\
$1000 & $50 \\
\end{array}
\]

We now illustrate three situations.

- Suppose first that the ongoing interest rate on the bond did not change during that year and the annual \( r_d \) remained at 10%. In that case:

\[
\text{Bond value at Year 1 if } r_d \text{ is } 10\% = 50(PVIFA)_{5\%,18} + 1000(PVIF)_{5\%,18}
\]

\[
= 50(11.690) + 1000(0.416)
\]

\[
= 584 + 416 = 1000
\]

Since \( r_d \) remained at 10%, which is the annual coupon rate, the bond continues to trade at par value of $1000.

- Suppose now that the ongoing interest rate on the bond increased during that year and now the annual \( r_d \) is 20%. In that case:

\[
\text{Bond value at Year 1 if } r_d \text{ is } 20\% = 50(PVIFA)_{10\%,18} + 1000(PVIF)_{10\%,18}
\]

\[
= 50(8.201) + 1000(0.180)
\]

\[
= 410 + 180 = 590
\]
Since the annual ongoing interest rate on the issuer given by $r_d$ increased to 20%, the bond now trades at a value below par, i.e., less than $1000. Such bond is called a discount bond.

- Suppose finally that the ongoing interest rate on the bond decreased during that year and now the annual $r_d$ is 6%. In that case:

\[
\text{Bond value at Year 1 if } r_d = 6\% = \$50(PVIFA)_{3\%,18} + $1000(PVIF)_{3\%,18}
\]
\[
= \$50(13.754) + $1000(0.587)
\]
\[
= 687 + 587 = $1274.
\]

Since the annual ongoing interest rate on the issuer (given by $r_d$) decreased to 6%, the bond now trades at a value above par, i.e., more than $1000. Such a bond is called a premium bond.

- **Example 1 (Semi-Annual Coupon Bond):** Suppose investors require an annual rate of 20% for holding a company’s bond. The company has a bond issue outstanding with 5 years to maturity with a coupon rate of 16 percent. Coupons are paid semiannually. What is the current value of these bonds? Set the par to $1000.

**Answer:**

\[
V_{bond} = $80(PVIFA)_{10\%,10} + $1000(PVIF)_{10\%,10}
\]
\[
= 491.6 + 386 = $877.6
\]

- **Example 2 (Buy-Hold and Sell):** Assume that you purchased a 1000 par value bond that pays a coupon of $100 every six months and has 5 years to go before it matures. The bond currently trades at par. If you buy this bond, you expect to hold it for one year and sell it in the market. You expect the market to require an annual rate of 16% when you sell the bond. How much should you expect receive for the bond when you sell it?

**Answer:**

\[
V_{bond \text{ at sell point}} = $100(PVIFA)_{8\%,8} + $1000(PVIF)_{8\%,8}
\]
\[
= $100(5.747) + $1000(0.54)
\]
\[
= $1114.7
\]
• **Example 3 (Semi-Annual Coupon Bond):** Suppose investors require an annual rate of 10% for holding a bond. The bond has 10 years to maturity with an annual coupon rate of 8 percent. Coupons are paid semiannually. What is the current value of this bond? Set the par to $1000.

**Answer:**

\[
V_{bond} = 40(PVIFA)_{5\%,20} + 1000(PVIF)_{5\%,20} \\
= 40(12.462) + 1000(0.377) = 875.48
\]

- **Example 4 (New Bond Issue):** IBM recently issued 20 year bonds at a price of $1000. These bonds pay $20 interest every 6 months. Their price remained stable since they were issued, i.e., they still sell at par. Due to additional financing needs, the company wants to issue new bonds that would have a maturity of 5 years, a par value of $1000 and pay $30 interest every 6 months. If the investors require the same annual rate of return as they require from the existing bonds, what would be the value of the new bonds?

**Answer:** First note that the required rate of return on the existing bonds is equal to the annual coupon rate on those bonds, since they trade at par. Accordingly, the annual ongoing rate that the investors require on the bond is given by

\[
Annual\ ongoing\ rate\ (r_d) = \frac{20}{1000} \times 2 = 4\%
\]

Now, we can compute the value of the new bonds as

\[
V_{bond} = 30(PVIFA)_{2\%,10} + 1000(PVIF)_{2\%,10} \\
= 30(8.983) + 1000(0.820) \\
= 1089.49
\]