Risk and Return

- Risk refers to the chance that some unfavorable event will occur. An asset’s risk can be analyzed in two ways.
  - on a stand-alone basis, where the asset is considered in isolation.
  - on a portfolio basis, where the asset is held as one of a number of assets in a portfolio.

- If an investor buys $100,000 of short term Treasury bills with a return of 5%, this investment is risk free. However, if the $100,000 were invested in the stock of a company just being organized to prospect for oil in mid-Atlantic, then the investment’s return could not be estimated precisely. In this case, there is a significant chance of actually earning much less than the expected return, and hence the stock investment is relatively risky.

An Illustrative Example

- To fix some ideas, and to be able to describe risk in a statistical sense, consider the following example.

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Sale.com Return</th>
<th>Basic Foods Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>0.3</td>
<td>100%</td>
<td>20%</td>
</tr>
<tr>
<td>Normal</td>
<td>0.4</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>Weak</td>
<td>0.3</td>
<td>-70%</td>
<td>10%</td>
</tr>
</tbody>
</table>

- Let us denote the expected rate of return by \( \hat{r} \) and first calculate the Sale.com’s expected rate of return. This is given by

\[
\hat{r}_{sale.com} = 0.3(100\%) + 0.4(15\%) + 0.3(-70\%) = 15\%
\]

Similarly, the expected rate of return for Basic Foods can be calculated as

\[
\hat{r}_{basic.foods} = 0.3(20\%) + 0.4(15\%) + 0.3(10\%) = 15\%
\]
• Both stocks have the same expected return of 15%, but Sale.com looks riskier. To formalize this intuition let us calculate the standard deviation of the two stocks.

• Standard Deviation (A Measure of Stand Alone Risk): For Basic Foods, the standard deviation (let us denote it by $\sigma$) is given by

$$
\sigma_{sale.com} = \sqrt{0.3(100\% - 15\%)^2 + 0.4(15\% - 15\%)^2 + 0.3(-70\% - 15\%)^2} \\
= 65.84\%
$$

For Sale.com, the standard deviation is given by

$$
\sigma_{basicfoods} = \sqrt{0.3(20\% - 15\%)^2 + 0.4(15\% - 15\%)^2 + 0.3(10\% - 15\%)^2} \\
= 3.87\%
$$

Sale.com has the larger standard deviation, which indicates a greater variation of returns, and thus a greater chance that the actual return may be substantially lower than the expected return. Therefore, Sale.com is a riskier investment than Basic Foods when held alone.

• Risk Premium: Risk averse investors demand a risk premium for bearing risk. Let us again illustrate this concept with the above example of Basic Foods and Sale.com. Suppose each stock sold for $100 per share, and each has an expected return of 15%. Since investors are risk averse, there will be a general preference for Basic Foods. Investors who hold Sale.com stock will start selling these shares and start buying Basic Food shares. As a result, Sale.com stock price will go down and Basic Foods stock price will go up. Suppose, for example, that, as a result of this shift, Basic Food stock price went up from $100 to $150, and the Sale.com stock price declined from $100 to $75. This means that (with these new prices), the expected return for Sale.com went up to 20%, whereas the expected return for Basic Foods went down to 10%. The difference in returns, 20% - 10% = 10%, is a risk premium, which represents the additional return that the investors demand to hold the riskier stock of Sale.com.
• **Risk in a Portfolio Context:** Let us now illustrate that an asset held in a portfolio is less risky than an asset held in isolation. Since most assets are held as part of portfolios, the risk and return of an individual security should be analyzed in terms of how that security affects the risk and return of the portfolio in which it is held. Consider the following two stocks in Table 2.

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Stock A Return</th>
<th>Stock B Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>0.3</td>
<td>100%</td>
<td>60%</td>
</tr>
<tr>
<td>Normal</td>
<td>0.4</td>
<td>15%</td>
<td>25%</td>
</tr>
<tr>
<td>Weak</td>
<td>0.3</td>
<td>-70%</td>
<td>10%</td>
</tr>
</tbody>
</table>

• First, let us compute the expected rate of returns and standard deviation for the two stocks when held in isolation. The expected returns are

\[
\hat{r}_A = 0.3(100\%) + 0.4(15\%) + 0.3(-70\%) = 15\%
\]

\[
\hat{r}_B = 0.3(60\%) + 0.4(25\%) + 0.3(10\%) = 31\%
\]

and the standard deviations are

\[
\sigma_A = \sqrt{0.3(100\% - 15\%)^2 + 0.4(15\% - 15\%)^2 + 0.3(-70\% - 15\%)^2}
= 65.84\%
\]

\[
\sigma_B = \sqrt{0.3(60\% - 31\%)^2 + 0.4(25\% - 31\%)^2 + 0.3(10\% - 31\%)^2}
= 19.97\%
\]
• Suppose these two stocks are held in a portfolio in equal weights, that is, both stocks account for 50% of the total portfolio value. In this case, we will have

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
<th>Portfolio Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>0.3</td>
<td>(0.5)100% + (0.5)60% = 80%</td>
</tr>
<tr>
<td>Normal</td>
<td>0.4</td>
<td>(0.5)15% + (0.5)25% = 20%</td>
</tr>
<tr>
<td>Weak</td>
<td>0.3</td>
<td>(0.5)(-70%) + (0.5)10% = -30%</td>
</tr>
</tbody>
</table>

Accordingly, expected portfolio return will be

\[ \hat{r}_{portfolio} = 0.3(80\%) + 0.4(20\%) + 0.3(-30\%) = 23\% \]

and the standard deviation of portfolio returns will be

\[ \sigma_{portfolio} = \sqrt{0.3(80\% - 23\%)^2 + 0.4(20\% - 23\%)^2 + 0.3(-30\% - 23\%)^2} \]

\[ \approx 42.67\% \]

• Another Example (Diversification): This example illustrates that two stocks, when held in isolation, can be quite risky, but when they are held together in a portfolio, they can diversify each other’s risk.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Stock A return</th>
<th>Stock B return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>80%</td>
<td>-90%</td>
</tr>
<tr>
<td>0.4</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>0.3</td>
<td>-70%</td>
<td>80%</td>
</tr>
</tbody>
</table>

The expected returns are

\[ \hat{r}_A = 0.3(80\%) + 0.4(10\%) + 0.3(-70\%) = 7\% \]

\[ \hat{r}_B = 0.3(-90\%) + 0.4(10\%) + 0.3(80\%) = 1\% \]

and the stand alone standard deviations are

\[ \sigma_A = \sqrt{0.3(80\% - 7\%)^2 + 0.4(10\% - 7\%)^2 + 0.3(-70\% - 7\%)^2} \]

\[ \approx 58.14\% \]
\[
\sigma_B = \sqrt{0.3(-90\% - 1\%)^2 + 0.4(10\% - 1\%)^2 + 0.3(80\% - 1\%)^2} \\
= 66.24\%
\]

Now consider a portfolio where these two stocks are held in equal weights, that is, both stocks account for 50% of the total portfolio value. In this case, we will have

<table>
<thead>
<tr>
<th>Probability</th>
<th>Portfolio return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.5(80%) + 0.5(-90%) = -5%</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5(10%) + 0.5(10%) = 10%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5(-70%) + 0.5(80%) = 5%</td>
</tr>
</tbody>
</table>

- Accordingly, expected portfolio return will be
  \[\bar{r}_{\text{portfolio}} = 0.3(-5\%) + 0.4(10\%) + 0.3(5\%) = 4\%\]

and the standard deviation of portfolio returns will be

\[
\sigma_{\text{portfolio}} = \sqrt{0.3(-5\% - 4\%)^2 + 0.4(10\% - 4\%)^2 + 0.3(5\% - 4\%)^2} \\
= 6.24\%
\]