Question 1: (Payoff From a Spread)

Consider the following spread created by call options on a stock:
A short position in a call option with a price of $3 and with a strike price of $20,
And a long position in a call option with a price of $1 and with a strike price of $30.
Both call options have the same expiration date.

a) (5 pts) What is the payoff to this spread if at expiration, stock price is $S_T=28$?

\[-(28-20) + 0 + 3 - 1 = -6\]

b) (5 pts) What is the payoff to this spread if at expiration, stock price is $S_T=40$?

\[-(40-20) + (40-30) + 3 - 1 = -8\]

c) (5 pts) What is the payoff to this spread if at the expiration date the stock price $S_T$ is between 20 and 30?

If $20 < S_T < 30 \implies -(S_T-20) + 3 - 1 \implies 22 - S_T$
Question 2: (Lower Bound on Put Option Price and Arbitrage Profit)

A European put option (on a stock) that expires in a year has a strike price of $33. The current stock price is $25 and the one-year risk-free interest rate is 10%. The price of this put is $3.

a) (5 points) Is arbitrage possible? What is the arbitrage position?

\[ \text{Lower Bound on put price} \rightarrow \frac{x}{1+r} - S_0 = \frac{33}{1.10} - 25 = $5 \]

Since \( p = $3 < \text{lower bound} \)

**Arbitrage Possible**

**Arbitrage Position**
- Buy the put at \( p = 3 \)
- Buy the stock at \( S_0 = 25 \)
- Borrow \( p + S_0 = $28 \text{ at } r = 10\% \)

b) (5 points) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 40 \)?

\[ \text{Payoff at } S_T = 40 \rightarrow 40 - 28 (1.10) = $9.2 \]

\[ \text{Payoff will be minimum if } S_T \leq 33 \]

\[ \text{Minimum Payoff} = 33 - 28 (1.10) = $2.2 \]
Question 3 (15 pts)

A trader holds 100 shares of IBM stock. The trader also has $10,000 in cash. Consider the following two strategies that the trader can follow.

Strategy 1: The trader holds the 100 shares for one year, and invests 10,000 cash in a risk free bond for an annual return of 5%.

\[ V_1 = 100 S_T + 10,000 \times (1.05) \]

\[ V_1 = 100 S_T + 10,500 \]

Strategy 2: The trader buys 100 put options on IBM with strike price X=200 that expire in one year. The price of each option is p=20. The trader then holds 100 IBM shares and invests the remaining cash in the risk free bond for an annual return of 5%.

For what values of IBM share price \( S_T \) in one year at the expiration date, does Strategy 2 prove to be the better one?

**Strategy 2 yields**

<table>
<thead>
<tr>
<th>( S_T &lt; 200 )</th>
<th>( S_T &gt; 200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 100 (200) + 8000 (1.05) )</td>
<td>( 100 S_T + 8000 (1.05) )</td>
</tr>
<tr>
<td>( = 28,400 )</td>
<td>( = 100 S_T + 8000 (1.05) )</td>
</tr>
</tbody>
</table>

**Strategy 2 is better if**

\[ 28,400 > 100 S_T + 10,500 \]

\[ 17900 > 100 S_T \quad \Rightarrow \quad S_T < 179 \]
Question 4: (20 points)

Consider the following portfolios:

Portfolio A: 20 long put options on IBM stock with strike price $80 and 100 shares of IBM stock. Each put gives the right to sell on share of the IBM stock.

Portfolio B: 20 short positions in call options on IBM stock with strike price $30 and some cash amount of $z invested at the risk free rate \( r = 10\% \). Each call gives the right to buy one share of the IBM stock.

Suppose that, if IBM stock price at expiration date turns out to be \( S_T = 60 \), then both portfolios has the same payoff.

What is the cash amount $z in portfolio B?

\[
\begin{array}{|c|c|}
\hline
\text{VALUE of A} & 20(80-60) + 100(60) \\
\hline
\text{VALUE of B} & -20(60-30) + 1.1z \\
\hline
\end{array}
\]

\[
20(20) + 100(60) = -20(30) + 1.1z
\]

\[
6400 = -600 + 1.1z
\]

\[
7000 = 1.1z
\]

\[
z = \frac{7000}{1.1}
\]
Question 5: (Arbitrage Profit when Call is Underpriced)

Consider a European call option (on a stock) that expires in one year. The strike price is $55. The current stock price is $60 and the one-year risk free interest rate is 10%. The price of this call is $7.

a) (5 points) Is arbitrage possible? What is the arbitrage position?

Lower Bound on \( C \) \( \Rightarrow \) \( S_0 - \frac{X}{1 + r} = 60 - \frac{55}{1.10} = 10 \)

Since \( C = 7 \leq \text{Lower Bound} \), arbitrage possible.

\[
\text{Arbitrage Position} \begin{cases} 
\text{Buy the call at } C = 7 \\
\text{Sell the stock short at } S_0 = 60 \\
\text{Invest } S_0 - C = 53 \text{ at } r = 10\% 
\end{cases}
\]

b) (5 points) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 70 \)?

\[
53 \left( 1.10 \right) - 55 = \$3.3
\]

c) (5 points) What is the minimum arbitrage profit in this situation?

Minimum profit when \( S_T > 55 \)

\[
\text{Minimum profit} = 53 \left( 1.10 \right) - 55
\]

\[= \$3.3\]
Question 6: (20 points)

Consider a trading position which involves

- A short position in a call option with a strike price $X = 80$ and a price $c = 10$.
- A short position in a put option with a strike price $X = 80$ and price $p = 10$.

Both options have the same underlying stock and the same expiration date.

Find and draw the payoff diagram for this position as a function of $S_T$

If $S_T < 80$ → $-(80 - S_T) + 10 + 10 = S_T - 60$

If $S_T > 80$ → $-(S_T - 80) + 10 + 10 = 100 - S_T$

[Diagram showing the payoff diagram with labeled points and the term "reverse straddle" highlighted.]