A model of international exchange

- How are exchange rates determined?
- What kind of international monetary system should be in place?
- Should trading partners fix their exchange rates?
- Should trading partners have a common currency?
- Consider two countries: $a$ and $b$ each with their own monies.
- Each country is modeled as in the previous chapters.
A model of international exchange

- Endowments in each country are of the same goods.
- People are indifferent to the origin of goods.
- Population growth rates: \( n^a \) and \( n^b \).
- Money supply growth rates: \( z^a \) and \( z^b \).
- All changes in money stock finance government spending.
- Assume free international trade.
- The exchange rate \( e_t \) is defined to be the units of country \( b \) money that can be purchased with one unit of country \( a \) money.
A model of international exchange

- Options of the owner of one unit of country $a$ money:
  1. Keep unit of country $a$ money. Buy $v_t^a$ goods.
  2. Trade for $e_t$ units of country $b$ money. Buy $e_t \cdot v_t^b$ goods.

- Options of the owner of one unit of country $b$ money:
  1. Keep unit of country $b$ money. Buy $v_t^b$ goods.
  2. Trade for $\frac{1}{e_t}$ units of country $a$ money. Buy $\frac{v_t^b}{e_t}$ goods.
A model of international exchange

- If \( v_t^a > e_t v_t^b \) everybody wants country a money. This cannot be an equilibrium where both monies are valued.

- If \( v_t^a < e_t v_t^b \) everybody wants country b money. This cannot be an equilibrium where both monies are valued.

- Only if \( v_t^a = e_t v_t^b \) can there be an equilibrium where both monies are valued:

\[
e_t = \frac{v_t^a}{v_t^b}.
\]

- We want to study the behavior of \( e_t \) under different international monetary arrangements.
Foreign currency controls

- Young agents can only take their country’s money to next period.

- Old agents can exchange their money to trade if they wish to (allow for international trade).

- Each county’s value of money is determined independently of the other. The exchange rate is:

  \[ e_t = \frac{v_t^a}{v_t^b} = \frac{N_t^a(y^a - c_1^{a,t})}{M_t^a} = \frac{N_t^a(y^a - c_1^{a,t})}{N_t^b(y^b - c_1^{b,t})} \frac{M_t^b}{M_t^a}. \]

- Exchange rate only depends on value of relative demand and relative supply.
Foreign currency controls

- From the previous chapter we know that: \[ \frac{v_{t+1}^a}{v_t^a} = \frac{n_a}{z_a} \] and \[ \frac{v_{t+1}^b}{v_t^b} = \frac{n_b}{z_b}. \]

- So the path of the exchange rate over time is given by:

\[
\frac{e_{t+1}}{e_t} = \frac{\frac{v_{t+1}^a}{v_{t+1}^b}}{\frac{v_t^a}{v_t^b}} = \frac{v_{t+1}^a}{v_t^a} \frac{v_t^b}{v_{t+1}^b} = \frac{n_a}{z_a} \frac{z_b}{n_b} = \frac{n_a}{n_b} \frac{z_b}{z_a}.
\]

- The greater the growth rate of country a’s population relative to country b’s, the greater the growth rate of the exchange rate (exchange rate appreciation.)

- The greater the growth rate of country a’s money supply relative to country b’s, the lower the growth rate of the exchange rate (exchange rate depreciation.)
Fixed exchange rates

- Suppose country $a$ wants to keep the exchange rate fixed: $e_{t+1} = e_t$. Then it must set:

$$z^a = \frac{n^a}{n^b}z^b.$$

- If it chooses to do so, then it can no longer print money freely to raise a chosen level of seignorage revenue.
- A country loses its monetary policy independence if it wants to pursue a fixed exchange rate policy.
- Given equal population growth rates, under fixed exchange rates, both countries will have the same rates of inflation.
- Foreign currency controls are costly because unnecessary exchanges are undertaken which are costly in the real world.