Increasing $M$ in growing economies

- Allow for growing population: $N_t = nN_{t-1}$.

- Rate of return on money:

\[
\frac{V_{t+1}}{V_t} = \frac{N_{t+1}(y-c_1)}{M_{t+1}} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} = \frac{n}{z}.
\]

- The budget set is now:

\[
c_1 + \left(\frac{Z}{n}\right) c_2 \leq y + \left(\frac{Z}{n}\right) a.
\]

- The feasible set is:

\[
N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y.
\]

\[
c_1 + \left(\frac{1}{n}\right) c_2 \leq y.
\]

- The monetary equilibrium is not optimal.
Golden rule in growing economies

- Should the money supply grow at the same rate as resources so as to keep the value of money constant?

- To maintain a constant value of money we need to set $z = n$, so that the stock of money expands at the same rate as the demand for money.

- The budget set becomes:

$$c_1 + c_2 \leq y + a$$

- Comparing this to the feasible set, we see that this monetary equilibrium is inefficient.

- Golden rule is to maintain fixed money stock.
Seignorage is the use of money creation in order to purchase goods:

\[ M_t - M_{t-1} = \left( 1 - \frac{1}{z} \right) M_t. \]

Goods acquired by the government:

\[ G_t = \left( 1 - \frac{1}{z} \right) v_t M_t. \]

Assume the use of goods acquired by government does not affect agents’ choices.

Lifetime budget constraint used to determine the monetary competitive equilibrium:

\[ c_1 + zc_2 \leq y. \]
Inefficiency of inflation as a tax

- Feasible set:
  \[ N_t c_{1,t} + N_{t-1} c_{2,t} + G_t \leq N_t y. \]

- Letting \( g = \frac{G_t}{N_{t-1}} \), we get:
  \[ c_1 + c_2 + g \leq y. \]

- The two sets do not coincide: the monetary equilibrium is not optimal.

- Inflation is not the best tax that can be used to finance government purchases.
A non-distorting tax

- Consider a **lump-sum** tax of $\tau$ goods collected from each old person.

- Budget constraints:
  
  $$c_{1,t} + v_t m_t \leq y.$$  
  $$c_{2,t+1} \leq v_{t+1} m_t - \tau_t.$$  

- This becomes (under stationarity):
  
  $$c_1 + c_2 \leq y - \tau.$$  

- Feasible set is the same as before:
  
  $$c_1 + c_2 \leq y - g.$$
Optimality of lump-sum taxation

- Government’s budget constraint:
  \[ G_t = N_{t-1} \tau_t. \]

- This implies \( \tau = g \).

- The sets coincide, therefore the equilibrium is optimal.

- By using lump-sum taxes the government was able to raise the necessary revenue without distorting the budget set (relative to the feasible set).

- Money creation is inferior to lump-sum taxation as a revenue device.

- Seignorage can be used to tax non-residents and illegal activities.
Limits to seignorage

- Government revenue is limited to the value (in terms of goods) of the money stock.

\[ M_t - M_{t-1} = \left( 1 - \frac{1}{z} \right) v_t M_t. \]

- Suppose government expenditure financed through lump-sum taxes and seignorage.

- As rate of inflation \((z)\) increases, tax rate increases: \((1 - \frac{1}{z})\).

- As rate of inflation \((z)\) increases, \((y - c_{1,t})\) decreases, so tax base decreases, but it is limited by size of desired real money balances: \(v_t M_t = N_t(y - c_{1,t}).\)

- Revenues from seignorage are increasing for low levels of \(z\), but decreasing for high levels.
Inflation in growing economies

Government spending

\( (n/z) \cdot y + a \)

\( c^G_R \)

\( c^m \)

\( y \)

\( y + (z/n) \cdot a \)
Inflation in growing economies

Government spending
Monetary equilibrium with seignorage
The inefficiency of an inflation tax