A growing supply of fiat money

- Focus on supply of fiat money.
- Consequences of increasing stock of fiat money.
- Use chapter 1’s model.
- Money supply: $M_t = zM_{t-1}$.
- $z > 1$ is gross rate of money supply growth.
- New money printed each period is:

$$M_t - M_{t-1} = M_t - \frac{M_t}{z} = \left(1 - \frac{1}{z}\right)M_t.$$
The government’s budget constraint

- How is money introduced in the economy?
- Through **lump sum** transfers to old consumers.
- **Government’s budget constraint:**

\[
N_{t-1} a_t = \left(1 - \frac{1}{z}\right) v_t M_t,
\]

\[
a_t = \frac{\left(1 - \frac{1}{z}\right) v_t M_t}{N_{t-1}}.
\]

- The lump sum transfer returns the new money to consumers, so that money stock expansion is not a transfer from consumers to government.
Individual budget constraints

- Budget constraints:
  \[ c_{1,t} + v_t m_t \leq y; \]
  \[ c_{2,t+1} \leq v_{t+1} m_t + a_{t+1}. \]

- Lifetime budget constraint:
  \[ c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq y + \frac{v_t}{v_{t+1}} a_{t+1}. \]

- Money market clearing:
  \[ v_t M_t = N_t(y - c_{1,t}). \]

- Stationarity implies:
  \[ v_t = \frac{N_t(y - c_1)}{M_t}. \]
Rate of return of fiat money

Rate of return of fiat money:

\[
\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y - c_1)}{M_{t+1}}}{\frac{N_t(y - c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{M_t}{zM_t} = \frac{1}{z}.
\]

If \( z > 1 \) the value of money declines over time.

The larger \( z \), the lower the rate of return.

Gross inflation rate

\[
\frac{p_{t+1}}{p_t} = \frac{1}{v_{t+1}} = \frac{v_t}{v_{t+1}} = z \Rightarrow p_{t+1} = zp_t.
\]

The price level increases over time at the same rate as the money stock.

Quantity Theory of Money holds: price levels are proportional to stocks of money.
Monetary equilibrium

- Substituting the rate of return of fiat money into the lifetime budget constraint:

\[ c_1 + zc_2 \leq y + za. \]

- The budget constraint is now different.

- The subsidies to the old do not come at zero cost (total resources are fixed).

- Money holders stand to lose since money loses value.

- Inflation works like a tax on money holdings that discourages money use.
Feasible set

- Is the equilibrium with inflation optimal?
- Compare it with possible alternatives.
- Compare budget set to feasible set.
- If they coincide, the golden rule allocation is attainable under the monetary equilibrium.
- Money stock expansion does not change feasible set.
- Feasible set still is:

\[ c_1 + c_2 \leq y. \]
Golden rule allocation

- The budget set is given by:

\[ c_1 + zc_2 \leq y + za. \]

- Point \((c_1^{GR}, c_2^{GR})\), which is feasible, yields higher utility for future generations than the monetary equilibrium \((c_1^m, c_2^m)\).

- Second period consumption is also higher than under the monetary equilibrium, so the initial old are also better off.

- The golden rule is not attainable because it is outside the budget set, the rate of return on money is too low to attain it.
Inefficiency of inflation

- Without money creation the two sets coincide \((z = 1, a = 0)\).

- What is the welfare cost of money creation?

- Agents still consume on the boundary of the feasible set, but consume relatively less of \(c_2\) since it requires the use of fiat money that has a relatively lower rate of return.

- Inflation taxes money holdings, therefore agents want to hold less money, which implies reducing the consumption of the goods acquired with money.

- The cost of inflation is that it drives people (needlessly) away from goods that require the use of money to be acquired.
Inflation