Financial Economics 4378  
Fall 2013  
Homework 1  
Due September 24th Tuesday  

There are 4 questions.  
Total Points is 100  

NAME:............................................

Question 1: (Lower Bound on Call Option Price and Arbitrage Profit)  
A European call option (on a stock) that expires in a year has a strike price of $66.  
The current stock price is $75 and the one-year risk free interest rate is 10%.  
The price of this call option is $8.  

a) Is arbitrage possible? What is the arbitrage position? (5 pts)  

Lower Bound on \( c \)  
\[
S_0 - \frac{X}{1+r} = 75 - \frac{66}{1.10} = 15  
\]

Since \( c = 8 < \) Lower Bound, Arbitrage Possible.  

\( \text{Arbitrage Position} = \begin{cases}  
\text{Buy the call at } c = 8 \\
\text{Sell the Stock Short at } S_0 = 75 \\
\text{Invest } S_0 - c = 75 - 8 = 67 \text{ at } r = 10\%  
\end{cases} \)

b) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 50 \)? (10 pts)  

\[
\text{Payoff} = 67(1.10) - 50 = 28.7  
\]

c) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 80 \)? (10 pts)  

\[
\text{Payoff} = 67(1.10) - 66 = 7.7  
\]
Question 2: (Arbitrage Profit when Put Price is Below its Lower Bound)

Consider a European put option (on a stock) that expires in one year. The strike price is $105. The current stock price is $90 and the one-year risk-free interest rate is 5%. The price of this put is $5.

a) Is arbitrage possible? What is the arbitrage position? (5 pts)

\[ \text{Lower Bound on } p = \frac{X - S_0}{1 + r} = \frac{105}{1.05} - 90 = 10 \]

Since \( p = 5 \leq \text{Lower Bound} = 10 \), arbitrage possible.

Arbitrage Position:
- Buy the put at \( p = 5 \)
- Buy the stock at \( S_0 = 90 \)
- Borrow \( p + S_0 = 95 \) at 5%.

b) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 130 \). (10 pts)

\[ \text{Payoff} = 130 - 95 \times (1.05) = 30.25 \]

c) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 80 \). (10 pts)

\[ \text{Payoff} = 105 - 95 \times (1.05) = 5.25 \]
Question 3: (25 points)

Consider a trading position which involves

A long position in a call option with a strike price $X = 90$ and a price $c = \$8$.
A long position in a put option with a strike price $X = 90$ and price $p = \$7$.

Both options have the same underlying stock and the same expiration date.

Find and draw the payoff diagram for this position as a function of $S_T$.

If $S_T < 90 \quad \Rightarrow \quad \text{call} + (90 - S_T) - \text{put} = 75 - S_T$

If $S_T > 90 \quad \Rightarrow \quad \text{call} + (S_T - 90) - \text{put} = S_T - 105$
Question 4: (Payoff From a Spread) (25 POINTS)

Consider the following spread created by call options on a stock:
A short position in a call option with a price of $4 and with a strike price of $30,
And a long position in a call option with a price of $2 and with a strike price of $40.
Both call options have the same expiration date.

a) (5 pts) State whether this is a bullish or a bearish position. Explain why.

   This is a bearish position as the call option sold has a lower strike price than the call option bought.

b) (10 pts) What is maximum loss that the trader can suffer in this position? For which values of \( S_T \) does this maximum loss occur?

   \[
   \text{For } S_T > 40 \rightarrow -(S_T - 30) + (S_T - 40) + 4 - 2 = -8 \quad \text{max loss for } S_T > 40
   \]

c) (10 pts) What is the payoff to this spread if at the expiration date the stock price \( S_T \) is between 20 and 30?

   \[
   \text{If } 20 < S_T < 30 \rightarrow -0 + 0 + 4 - 2 = \pm 2
   \]

\[\text{Payoff}\]

\[\begin{array}{c|c}
\text{Payoff} & 2 \quad 32 \quad 40 \\
\hline
-8 & 0 & \end{array}\]

\[\text{Stock Price } S_T\]