ECO 5341 Wages and Employment in a Unionized Firm

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Consider a monopolist firm in a product market. The firm faces an inverse demand function

\[ P = 150 - q \]

where \( q \) is the firm’s output and \( P \) is the price.
Labor Demand by the Firm

- The firm only uses labor as an input. Suppose that to produce one unit of output, the firm needs one unit of labor. Hence, we have

$$q = L$$

where $L$ is the total labor input the firm uses.

- Each unit of labor costs the firm a wage $w$. The firm’s profit function is thus given by

$$\pi(L, w) = (150 - L)L - wL$$
The Union

- While the Firm has exclusive control over how much labor to hire, there is a Union that has exclusive control over the wage $w$.
- The Union chooses $w$ to maximize

$$U(w, L) = (w - w_a)L$$

where $w_a$ is the minimum wage that the workers can secure themselves in an alternative employment.
- The Union’s payoff increasing both in wage $w$ (chosen by the Union) and by $L$ (chosen by the Firm).
Sequence of moves in the game

In the first stage, the Union chooses $w$ to maximize

$$U(w, L) = (w - w_a)L = wL - w_aL$$

In the second stage, after observing $w$ set by the Union, the Firm chooses $L$ to maximize profits given by

$$\pi(L, w) = (150 - L)L - wL$$
Backward Induction Equilibrium

- Let’s first solve the Firm’s best response function after the Union sets $w$.
- Given the wage $w$ set by the Union, the Firm chooses $L$ to maximize profits given by

$$\pi(L, w) = (150 - L)L - wL = 150L - L^2 - wL$$

- First order condition yields

$$150 - 2L - w = 0 \Rightarrow L^*(w) = 75 - \frac{w}{2}$$
The Union’s Problem

In the first stage, anticipating the Firm’s subsequent best response $L^*(w)$, the Union chooses $w$ to maximize

$$U(w, L^*(w)) = (w - w_a)L^*(w) = (w - w_a)\left(75 - \frac{w}{2}\right)$$

Therefore, the Union chooses $w$ to maximize

$$U(w, L^*(w)) = 75w - \frac{w^2}{2} - 75w_a + \frac{w_aw}{2}$$
The first order condition is

$$75 - w^* + \frac{w_a}{2} = 0 \Rightarrow w^* = 75 + \frac{w_a}{2}$$

How does optimal wage $w^*$ depend on minimum wage $w_a$?
Now let’s go back to the firm’s best response in stage 2 to find $L^*$. We have

$$L^*(w) = 75 - \frac{w}{2} \quad w^* = 75 + \frac{w_a}{2}$$

Hence

$$L^*(w^*) = 75 - \frac{1}{2} \left( 75 + \frac{w_a}{2} \right)$$

$$\Rightarrow L^* = \frac{75}{2} - \frac{w_a}{4}$$

How does optimal employment $L^*$ depend on minimum wage $w_a$?