ECO 5341 Stackelberg Model of Duopoly

Saltuk Ozerturk (SMU)
Stackelberg Duopoly

- Suppose that two firms (Firm 1 and Firm 2) face an industry demand

\[ P = 150 - Q \]

where

\[ Q = q_1 + q_2 \]

is the total industry output. Both firms have the same unit production cost \( c = 30 \).

- Assume that first Firm 1 moves and chooses \( q_1 \). In the second stage, after observing \( q_1 \), Firm 2 moves and chooses \( q_2 \).
Deriving Firm 2’s best response in the second stage

Given \( q_1 \) chosen by Firm 1, Firm 2 chooses \( q_2 \) to maximize

\[
\pi_1 (q_1, q_2) = (150 - q_1 - q_2)q_2 - 30q_2
\]

First order condition:

\[
150 - 2q_2 - q_1 - 30 = 0
\]

which yields the best response function:

\[
q_2^*(q_1) = 60 - \frac{q_1}{2}
\]
Deriving Firm 1’s optimal choice of $q_1$

- Note that Firm 1 perfectly knows how Firm 2 will respond to any $q_1$ that it chooses. Firm 1 chooses $q_1$ to maximize

$$\pi_1(q_1, q_2) = (150 - q_1 - q_2^*(q_1))q_1 - 30q_1$$

subject to

$$q_2^*(q_1) = 60 - \frac{q_1}{2}$$
Firm 1 chooses $q_1$ to maximize

$$\pi_1 (q_1, q_2) = (150 - q_1 - q_2^*(q_1))q_1 - 30q_1$$

which becomes

$$\pi_1 (q_1, q_2) = (150 - q_1 - \left(60 - \frac{q_1}{2}\right))q_1 - 30q_1$$

$$= 90q_1 - \frac{q_1^2}{2} - 30q_1$$

and yields the first order condition

$$90 - q_1^S - 30 = 0 \Rightarrow q_1^S = 60$$

$$\Rightarrow q_2^S = 60 - \frac{q_1^S}{2} \Rightarrow q_2^S = 60 - \frac{60}{2} \Rightarrow q_2^S = 30$$
Comparison of Cournot Duopoly NE and Stackelberg NE

<table>
<thead>
<tr>
<th></th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$P$</th>
<th>$\pi_1^*$</th>
<th>$\pi_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cournot NE</strong></td>
<td>40</td>
<td>40</td>
<td>70</td>
<td>1600</td>
<td>1600</td>
</tr>
<tr>
<td><strong>Stackelberg NE</strong></td>
<td>60</td>
<td>30</td>
<td>60</td>
<td>1800</td>
<td>900</td>
</tr>
</tbody>
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- Firm 1 who moves first enjoys a first mover advantage. Behaves more aggressively and secures more profits $\pi_1^*$ than what it achieves in Cournot NE.