EXAMPLE 3
Consider an overlapping generations economy, where

- When young, a consumer has
  \[ y = 240 \]
  and has no endowment when old.
- Population is constant, i.e. we have
  \[ N_t = N_{t-1} \]
- The slope of a consumer’s indifference curve is
  \[ \frac{-c_2}{3c_1} \]
  where \( c_1 \) is consumption when young and \( c_2 \) is consumption when old. Assume stationarity.
- The stock of money supply is growing at a rate \( z = 1.2 \), i.e.,
  \[ M_t = (1.20)M_{t-1} \]
- Suppose that this newly printed money is distributed equally among the old of that period. That is, the old people in any period \( t \) receive a transfer \( a_t \) measured in terms of the consumption good.

a) Find the golden rule allocation \((c_1^{GR}, c_2^{GR})\) that a social planner would choose.

**ANSWER:** The social planner’s budget constraint is

\[ N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y \]

which can be rewritten as

\[ c_{1,t} + \frac{N_{t-1}}{N_t} c_{2,t} \leq y \rightarrow c_{1,t} + c_{2,t} \leq 240 \]

with stationarity, this becomes

\[ c_1 + c_2 \leq 240 \]

Note that the slope of the budget line is

\[ -1. \]

Golden rule allocation is defined by two conditions
• (i) It must be on the budget line, i.e.,

\[ c_1^{GR} + c_2^{GR} = 240 \]  \hspace{1cm} (1)

(ii) The slope of the indifference curve at the golden rule allocation must be equal to the slope of the budget line, i.e.,

\[
-1 = \frac{c_2^{GR}}{3c_1^{GR}}
\]

\[ \Rightarrow 3c_1^{GR} = c_2^{GR} \]  \hspace{1cm} (2)

Combining (1) and (2), we get

\[ c_1^{GR} + 3c_1^{GR} = 240 \]

\[ \Rightarrow 4c_1^{GR} = 240 \]

\[ \Rightarrow c_1^{GR} = 60 \text{ and } c_2^{GR} = 180. \]

b) Suppose a consumer of generation \( t \) can buy \( m_t \) units money when young and use the money to finance consumption when old. Let the value of money at period \( t \) be denoted by \( v_t \). Write down two budget constraints when young and old and combine them to obtain a lifetime budget constraint

**ANSWER:** Budget constraint when young

\[ c_{1,t} + v_t m_t \leq 240 \]  \hspace{1cm} (3)

Budget constraint when old

\[ c_{2,t+1} \leq v_{t+1} m_t + a_{t+1} \]  \hspace{1cm} (4)

Rewrite the young age budget constraint as

\[ \frac{c_{1,t}}{v_t} + m_t \leq \frac{240}{v_t} \]  \hspace{1cm} (5)

and rewrite the old age budget constraint as

\[ \frac{c_{2,t+1}}{v_{t+1}} \leq m_t + \frac{a_{t+1}}{v_{t+1}} \]  \hspace{1cm} (6)

Now sum up (5) and (6) to obtain the lifetime budget constraint of generation \( t \) as

\[ \frac{c_{1,t}}{v_t} + \frac{c_{2,t+1}}{v_{t+1}} \leq \frac{240}{v_t} + \frac{a_{t+1}}{v_{t+1}} \]

\[ \Rightarrow c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 240 + \frac{v_t a_{t+1}}{v_{t+1}} \]

c) Find the equilibrium rate of return on money \( \frac{v_{t+1}}{v_t} \) by equating demand for money at period \( t \) to supply of money at period \( t \) and assuming stationarity.
**Answer:** Demand for money at time \( t \) is 
\[ N_t(y - c_{1,t}) \]
Supply of money at time \( t \) is 
\[ v_t M_t \]
Equating supply and demand, we get 
\[ v_t M_t = N_t(y - c_{1,t}) \Rightarrow v_t = \frac{N_t(y - c_{1,t})}{M_t} \]
Similarly for period \( t+1 \), we have 
\[ v_{t+1} = \frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}} \]
As a result, we obtain 
\[ \frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}}}{\frac{N_t(y - c_{1,t})}{M_t}} \Rightarrow \frac{v_{t+1}}{v_t} = \frac{N_{t+1} M_t}{N_t M_{t+1}} \frac{(y - c_{1,t+1})}{(y - c_{1,t})} \]
But with stationarity, we have \( c_{1,t+1} = c_{1,t} \) and we have constant population, i.e. 
\[ \frac{N_{t+1}}{N_t} = 1 \]
Therefore the above expression simplifies to 
\[ \frac{v_{t+1}}{v_t} = \frac{M_t}{M_{t+1}} = \frac{1}{\frac{1}{20}} = 1.20 \]
d) Revisit the individual lifetime budget constraint you found in part (b) and rewrite it using the equilibrium rate of return on money you found in part (c). How does the individual lifetime budget constraint differ from the social planner’s budget constraint in (a). Is the competitive monetary equilibrium allocation different from the golden rule allocation?

**Answer:** The individual lifetime budget constraint we found in part (b) was 
\[ c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 240 + \frac{v_t a_{t+1}}{v_{t+1}} \]
Imposing 
\[ \frac{v_{t+1}}{v_t} = \frac{1}{1.20} \]
this becomes 
\[ c_{1,t} + 1.2c_{2,t+1} \leq 240 + 1.2a \]
But note that with stationarity, this becomes 
\[ c_1 + 1.2c_2 \leq 240 + 1.2a \]
which is **DIFFERENT** than social planner’s budget constraint. Therefore we arrive at the conclusion that the competitive monetary equilibrium allocation **IS NOT THE SAME AS** the golden rule allocation when money supply is growing.