EXAMPLE 2
Consider the SAME ECONOMY as in Example 1, but this time suppose that the money supply is growing at a rate \( z = 1.1 \). In particular, we again have

- When young, a consumer has
  \[ y = 100 \]
  and has no endowment when old.
- Population grows at a constant rate \( n = 1.20 \), i.e. we have
  \[ \frac{N_t}{N_{t-1}} = n = 1.20 \]
- The slope of a consumer’s indifference curve is
  \[ -\frac{c_2}{c_1} \]
  where \( c_1 \) is consumption when young and \( c_2 \) is consumption when old. Assume stationarity.
- The stock of money supply is growing at a rate \( z = 1.1 \), i.e.,
  \[ M_{t+1} = (1.10)M_t \]
- Suppose that this newly printed money is distributed equally among the old of that period. That is, the old people in any period \( t \) receive a transfer \( a \) measured in terms of the consumption good.
  
  a) Find the golden rule allocation \( (c_1^{GR}, c_2^{GR}) \) that a social planner would choose.

  ANSWER: The social planner’s budget constraint is exactly the same as in Example 1, therefore the golden rule allocation is again \( c_2^{GR} = 60 \) and \( c_1^{GR} = 50 \).

  b) Suppose a consumer of generation \( t \) can buy \( m_t \) units money when young and use the money to finance consumption when old. Let the value of money at period \( t \) be denoted by \( v_t \). Write down two budget constraints when young and old and combine them to obtain a lifetime budget constraint

  ANSWER: Budget constraint when young
  \[ c_{1,t} + v_t m_t \leq 100 \quad (1) \]
  Budget constraint when old
  \[ c_{2,t+1} \leq v_{t+1} m_t + a \quad (2) \]
Rewrite the young age budget constraint as
\[ \frac{c_{t,1}}{v_t} + m_t \leq \frac{100}{v_t} \]  
(3)
and rewrite the old age budget constraint as
\[ \frac{c_{t+1,2}}{v_{t+1}} \leq m_t + \frac{a}{v_{t+1}} \]  
(4)
Now sum up (3) and (4) to obtain the lifetime budget constraint of generation t as
\[ \frac{c_{1,t}}{v_t} + \frac{c_{2,t+1}}{v_{t+1}} \leq \frac{100}{v_t} + \frac{a}{v_{t+1}} \]
\[ \Rightarrow c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 100 + \frac{v_t a}{v_{t+1}} \]
c) Find the equilibrium rate of return on money \( \frac{v_{t+1}}{v_t} \) by equating demand for money at period t to supply of money at period t and assuming stationarity.

**ANSWER:** Demand for money at time t is \( N_t(y - c_{1,t}) \)

Supply of money at time t is \( v_t M_t \)

Equating supply and demand, we get
\[ v_t M_t = N_t(y - c_{1,t}) \implies v_t = \frac{N_t(y - c_{1,t})}{M_t} \]

Similarly for period t+1, we have
\[ v_{t+1} = \frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}} \]

As a result, we obtain
\[ \frac{v_{t+1}}{v_t} = \frac{N_{t+1}(y - c_{1,t+1})}{N_t(y - c_{1,t})} \]
\[ \Rightarrow \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} \frac{(y - c_{1,t+1})}{(y - c_{1,t})} \]

But with stationarity, we have \( c_{1,t+1} = c_{1,t} \), therefore the above expression simplifies to
\[ \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} \]

which implies that equilibrium rate of return on money is given by
\[ \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} = \frac{1.20}{1.10} = 1.09 \]
d) Revisit the individual lifetime budget constraint you found in part (b) and rewrite it using the equilibrium rate of return on money you found in part (c). How does the individual lifetime budget constraint differ from the social planner’s budget constraint in (a). Is the competitive monetary equilibrium allocation different from the golden rule allocation?

**Answer:** The individual lifetime budget constraint we found in part (b) was

\[ c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 100 + \frac{v_t a}{v_{t+1}} \]

Imposing

\[ \frac{v_{t+1}}{v_t} = 1.09 \]

this becomes

\[ c_{1,t} + \frac{1}{1.09} c_{2,t+1} \leq 100 + \frac{a}{1.09} \]

or

\[ 10.9c_{1,t} + 10c_{2,t+1} \leq 1090 + 10a \]

But note that with stationarity, this is the same as

\[ 10.9c_1 + 10c_2 \leq 1090 + 10a \]

which is DIFFERENT than social planner’s budget constraint. **Therefore we arrive at the conclusion that the competitive monetary equilibrium allocation IS NOT THE SAME AS** the golden rule allocation when money supply is growing.