EXAMPLE
Consider the following overlapping generations economy studied in class.

- When young, a consumer has
  \[ y = 100 \]
  and has no endowment when old.
- Population grows at a constant rate \( n = 1.20 \), i.e. we have
  \[ \frac{N_t}{N_{t-1}} = n = 1.20 \]
- The slope of a consumer’s indifference curve is
  \[ \frac{c_2}{c_1} \]
  where \( c_1 \) is consumption when young and \( c_2 \) is consumption when old. Assume stationarity.
- The stock of money supply is constant in the economy, i.e.,
  \[ M_{t+1} = M_t = M \]

a) **Find the golden rule allocation** \((c_1^{GR}, c_2^{GR})\) that a social planner would choose.

**ANSWER:** The social planner’s budget constraint is
\[ N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y \]
which can be rewritten as
\[ c_{1,t} + \frac{N_{t-1}}{N_t} c_{2,t} \leq y \rightarrow c_{1,t} + \frac{1}{1.2} c_{2,t} \leq 100 \]
\[ \rightarrow 12c_{1,t} + 10c_{2,t} \leq 1200 \]
With stationarity, this becomes
\[ 12c_1 + 10c_2 \leq 1200 \]
Note that the slope of the budget line is
\[ -1.2 \]
Golden rule allocation is defined by two conditions
• (i) It must be on the budget line, i.e.,

$$12c_{1}^{GR} + 10c_{2}^{GR} = 1200 \quad (1)$$

(ii) The slope of the indifference curve at the golden rule allocation must be equal to the slope of the budget line, i.e.,

$$-1.2 = -\frac{c_{2}^{GR}}{c_{1}^{GR}}$$

$$\implies 12c_{1}^{GR} = 10c_{2}^{GR} \quad (2)$$

Combining (1) and (2), we get

$$12c_{1}^{GR} + 10c_{2}^{GR} = 1200$$

$$\implies 10c_{2}^{GR} + 10c_{2}^{GR} = 1200$$

$$\implies c_{2}^{GR} = 60 \text{ and } c_{1}^{GR} = 50.$$  

• b) Suppose a consumer of generation $t$ can buy $m_t$ units of money when young and use the money to finance consumption when old. Let the value of money at period $t$ be denoted by $v_t$. Write down two budget constraints when young and old and combine them to obtain a lifetime budget constraint

**Answer:** Budget constraint when young

$$c_{1,t} + v_t m_t \leq 100 \quad (3)$$

Budget constraint when old

$$c_{2,t+1} \leq v_{t+1} m_t \quad (4)$$

Rewrite the young age budget constraint as

$$\frac{c_{1,t}}{v_t} + m_t \leq \frac{100}{v_t} \quad (5)$$

and rewrite the old age budget constraint as

$$\frac{c_{2,t+1}}{v_{t+1}} \leq m_t \quad (6)$$

Now sum up (5) and (6) to obtain the lifetime budget constraint of generation $t$ as

$$\frac{c_{1,t}}{v_t} + \frac{c_{2,t+1}}{v_{t+1}} \leq \frac{100}{v_t}$$

$$\implies c_{1,t} + v_t \frac{c_{2,t+1}}{v_{t+1}} \leq 100.$$
c) Find the equilibrium rate of return on money \( \frac{v_{t+1}}{v_t} \) by equating demand for money at period \( t \) to supply of money at period \( t \) and assuming stationarity.

**ANSWER:** Demand for money at time \( t \) is

\[ N_t(y - c_{1,t}) \]

Supply of money at time \( t \) is

\[ v_t M_t \]

Equating supply and demand, we get

\[ v_t M_t = N_t(y - c_{1,t}) \implies v_t = \frac{N_t(y - c_{1,t})}{M_t} \]

Similarly for period \( t+1 \), we have

\[ v_{t+1} = \frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}} \]

As a result, we obtain

\[ \frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}}}{\frac{N_t(y - c_{1,t})}{M_t}} \]

\[ \Rightarrow \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} \frac{(y - c_{1,t+1})}{(y - c_{1,t})} \]

But with stationarity, we have \( c_{1,t+1} = c_{1,t} \), therefore the above expression simplifies to

\[ \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} \]

Also, in this question, we are told that money supply is constant and hence

\[ M_{t+1} = M_t = M \]

which implies that equilibrium rate of return on money is given by

\[ \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} = 1.20 \]

d) Revisit the individual lifetime budget constraint you found in part (b) and rewrite it using the equilibrium rate of return on money you found in part (c). How does the individual lifetime budget constraint differ from the social planner’s budget constraint in (a). Is the competitive monetary equilibrium allocation different from the golden rule allocation?

**Answer:** The individual lifetime budget constraint we found in part (b) was

\[ c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 100 \]
Imposing \[
\frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} = 1.20
\]
this becomes
\[
c_{1,t} + \frac{1}{1.2} c_{2,t+1} \leq 100
\]
or
\[
12c_{1,t} + 10c_{2,t+1} \leq 1200
\]
But note that with stationarity, this is the same as
\[
12c_1 + 10c_2 \leq 1200
\]
which is exactly the same as the social planner’s budget constraint. Therefore we arrive at the conclusion that the competitive monetary equilibrium allocation IS THE SAME AS the golden rule allocation.