ECO 5341 Cournot Competition and Collusion

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Cournot Competition

Suppose that two firms (Firm 1 and Firm 2) face an industry demand

\[ P = 150 - Q \]

where

\[ Q = q_1 + q_2 \]

is the total industry output.

Both firms have the same unit production cost \( c = 30 \).

The firms are competing by simultaneously setting their quantities to maximize own profits.
Cournot Competition

Deriving Firm 1’s best response

For any given $q_2$, Firm 1 chooses $q_1$ to maximize

$$\pi_1(q_1, q_2) = (150 - q_1 - q_2)q_1 - 30q_1$$

First order condition:

$$150 - 2q_1 - q_2 - 30 = 0$$

which yields the best response function:

$$q_1^*(q_2) = 60 - \frac{q_2}{2}$$
Deriving Firm 2’s best response

For any given $q_1$, Firm 2 chooses $q_2$ to maximize

$$\pi_2(q_1, q_2) = (150 - q_1 - q_2)q_2 - 30q_2$$

First order condition:

$$150 - 2q_2 - q_1 - 30 = 0$$

which yields the best response function:

$$q_2^*(q_1) = 60 - \frac{q_1}{2}$$
Cournot Nash Equilibrium Pair Solves $q_1^C$ and $q_2^C$ solve

\[
q_1^C(q_2^C) = 60 - \frac{q_2^C}{2}
\]

\[
q_2^C(q_1^C) = 60 - \frac{q_1^C}{2}
\]

which yields

\[q_1^C = q_2^C = 40\]
Cournot Equilibrium continued

The equilibrium market price in the Cournot NE is given by

\[ P^c = 150 - Q = 150 - (q_1^c + q_2^c) = 70 \]
Cournot Profits of Each Firm

\[ \pi_1^c (q_1^c, q_2^c) = (150 - q_1^c - q_2^c)q_1^c - 30q_1^c \]
\[ \Rightarrow \quad \pi_1 (q_1, q_2) = (150 - 80) \times 40 - 30 \times 40 = 1600 \]

\[ \pi_2^c (q_1^c, q_2^c) = (150 - q_1^c - q_2^c)q_2^c - 30q_2^c \]
\[ \Rightarrow \quad \pi_2 (q_1, q_2) = (150 - 80) \times 40 - 30 \times 40 = 1600 \]
Monopoly Output and Price in this market is (how do I know?)

\[ q^m = 60 \]

\[ P^m = 150 - Q = 150 - 60 = 90 \]

which yields a monopoly profit

\[ \pi^m = (P^m - c)q^m = (90 - 30) \times 60 = 3600 \]
In principle, these two firms could decide to collude and act like a monopolist, each agreeing to produce half of the monopoly output \((q_1 = q_2 = q^m/2 = 30)\) and sell at the monopoly price \(P^m = 90\), making a profit of 1800 each, which is better than Cournot profit of \(\pi_1^C = \pi_2^C = 1600\).

But is collusion \(q_1 = q_2 = q^m/2 = 30\) a Nash Equilibrium?
Collusion outcome \( q_1 = q_2 = q^m/2 = 30 \) is not a Nash Equilibrium.

- Suppose Firm 2 produces at

\[
q_2 = q^m/2 = 30
\]

- What is Firm 1’s best response to \( q_2 = q^m/2 = 30 \)?

\[
q_1^*(q_2) = 60 - \frac{q_2}{2} = 60 - \frac{30}{2} = 45
\]
A Finite (Baby) version of Collusion Game. This is Exercise 1.5 in the textbook (page 49)

Suppose each of the two firms can either produce \( q^m/2 = 30 \) or the Cournot equilibrium quantity \( q^c = 40 \). No other quantity is feasible. The game matrix looks like this (you should verify all the payoffs)

<table>
<thead>
<tr>
<th></th>
<th>( q^m/2 = 30 )</th>
<th>( q^c = 40 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q^m/2 = 30 )</td>
<td>(1800, 1800)</td>
<td>(1500, 2000)</td>
</tr>
<tr>
<td>( q^c = 40 )</td>
<td>(2000, 1500)</td>
<td>(1600, 1600)</td>
</tr>
</tbody>
</table>

But note that this game is a Prisoners’ Dilemma! \( q^c = 40 \) is a strictly dominant action for both players.