ECO 5341 An Example of a Contribution Game

Saltuk Ozerturk (SMU)

January 2016
An Example of a Contribution Game

Two roommates Robert and Larry each want to choose how much time to allocate to keep the kitchen clean.

If Robert allocates $x_R$ hours and Larry allocates $x_L$ hours for kitchen cleaning their payoffs are

$$\pi_R(x_R, x_L) = 100x_R - 10x_Rx_L - 10x_R^2$$ (for Robert)

$$\pi_L(x_R, x_L) = 100x_L - 10x_Rx_L - 10x_L^2$$ (for Larry)

Find the Nash equilibrium contributions $x_R^*$ and $x_L^*$ of each player to keep the kitchen clean.
An Example of a Contribution Game

Deriving Robert’s best response function

Given any $x_L$ by Larry, Robert chooses $x_R$ to maximize

$$\pi_R(x_R, x_L) = 100x_R - 10x_Rx_L - 10x_R^2$$

First Order Condition yields

$$100 - 10x_L - 20x_R = 0$$

$$\implies x_R^*(x_L) = 5 - \frac{x_L}{2}$$ is Robert’s best response.
An Example of a Contribution Game

Deriving Larry's best response function

- Given any $x_R$ by Robert, Larry chooses $x_L$ to maximize

$$
\pi_L(x_R, x_L) = 100x_L - 10x_Rx_L - 10x_L^2
$$

First Order Condition yields

$$
100 - 10x_R - 20x_L = 0
$$

$$
x_L^*(x_R) = 5 - \frac{x_R}{2}
$$

is Larry's Best Response
An Example of a Contribution Game

The best response functions are

\[ x_R^*(x_L) = 5 - \frac{x_L}{2} \quad \text{and} \quad x_L^*(x_R) = 5 - \frac{x_R}{2} \]

- The Nash Equilibrium pair \((x_R^*, x_L^*)\) solves the above pair of equations.

\[ x_R^* = 5 - \frac{x_L^*}{2} \]

\[ \Rightarrow x_R^* = 5 - \frac{1}{2} \left( 5 - \frac{x_R^*}{2} \right) \]

\[ \Rightarrow x_R^* = \frac{5}{2} + \frac{x_R^*}{4} \Rightarrow 3x_R^* = 10 \Rightarrow x_R^* = \frac{10}{3} \]

\[ x_L^*(x_R^*) = 5 - \frac{x_R^*}{2} = 5 - \frac{1}{2} \left( \frac{10}{3} \right) \Rightarrow x_L^* = \frac{10}{3} \]