ECO 5341  
Spring 2016  
Homework 6 Solutions  
Total Points: 100

This homework assignment will only be accepted if the answers are provided on the space following each question. Treat this assignment like an exam. Write your answers only on the space provided following each question. Do not use separate sheets. Do not write your answers on other sheets of paper. For full credit, please be concise and tidy. If your answer is illegible and not well organized, you will lose points!

**Question 1 (25 points):** A professor has assigned a homework to a student, who could either plagiarize (P) or be honest (H). The professor could either check if the student has plagiarized (C) or choose not to check (N). The professor is either “soft” or “tough” and the payoff matrices (where student is the row player and the professor is the column player) corresponding to the two types are given as follows:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>(0,1)</td>
<td>(2,0)</td>
</tr>
<tr>
<td>H</td>
<td>(1,2)</td>
<td>(1,3)</td>
</tr>
</tbody>
</table>

Professor is Soft Type S (with prob 1-\( q \))

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>P</td>
<td>(0,1)</td>
<td>(2,0)</td>
</tr>
<tr>
<td>H</td>
<td>(1,4)</td>
<td>(1,3)</td>
</tr>
</tbody>
</table>

Professor is Tough Type T (with prob \( q \))

Find all pure strategy Bayesian Equilibria of the above game.
Answer: First note that for Type T playing C is a strictly dominant strategy. Hence, in any equilibrium we must have Type T playing C with probability 1.

Therefore, there are two possible equilibria.

- A pooling equilibrium in which both Type S and Type T play C.
- A separating equilibrium in which Type T plays C and Type S play N.

Let’s analyze these possible equilibria.

**A pooling equilibrium: Type S and Type T both play C.**

- Let’s consider the payoff to Player 1 (Student) in this pooling equilibrium. If Student plays P, he gets

  \[ q.0 + (1 - q).0 = 0 \]

  If Student plays H, he gets

  \[ q.1 + (1 - q).1 = 1 \]

  Hence, when both Type T and Type S play C, the student behaves honestly and plays H for all q. But note that when student plays H, Type S professor plays N instead of C. Hence we do not have a pooling equilibrium in which **Type S and Type T both play C.**

- **Conclusion:** The above game has no Pooling Equilibrium.

**Separating equilibrium: Type T plays C and Type S play N**

- First note that for Type S playing N to be a best response, Player 1 (Student) must be playing H. Therefore, we need to describe when the Student plays H.

- If Student plays H, he gets

  \[ (1 - q).1 + q.1 = 1 \]

  If Student plays P, he gets

  \[ (1 - q).2 + q.0 = 2 - 2q \]

  Hence, Student plays H if and only if

  \[ 1 \geq 2 - 2q \Rightarrow 2q \geq 1 \Rightarrow q \geq \frac{1}{2} \]
• Conclusion: For $q \geq \frac{1}{2}$, the above game has a Separating equilibrium in which
  
  – Type T plays C.
  – Type S play N.
  – Player 1 (Student) plays H.
Question 2 (25 points)

Consider the following Cournot Duopoly under Asymmetric Information. Suppose that two firms (Firm 1 and Firm 2) face an industry demand

\[ P = 600 - Q \]

where \( Q = q_1 + q_2 \) is the total industry output. Firm 1 has a unit production cost \( c_1 = \$60 \).

Firm 2 is either a high cost type or a low cost type. Firm 2’s cost function is \( c^H_2 = \$90 \) with probability \( \theta \) and \( c^L_2 = \$30 \) with probability \( 1 - \theta \)

- Firm 2 knows its own cost function and also knows Firm 1’s cost function.
- Firm 1 knows its own cost function but believes \( c^H_2 = \$90 \) with probability \( \theta \) and \( c^L_2 = \$30 \) with probability \( 1 - \theta \).

Find the Bayesian Equilibrium quantities \( q_1, q_H \) and \( q_L \) as a function of \( \theta \).
Deriving Firm 2 of Type H’s (high cost) best response

• For any given $q_1$, Firm 2 of Type H chooses $q_H$ to maximize

$$\pi_2(q_1, q_L) = (600 - q_1 - q_H)q_H - 90q_H$$

First order condition:

$$600 - 2q_H - q_1 - 90 = 0$$

which yields the best response function:

$$q_H^*(q_1) = 255 - \frac{q_1}{2}$$ (BR$_H$)

Deriving Firm 2 of Type L’s (low cost) best response

• For any given $q_1$, Firm 2 of Type L chooses $q_L$ to maximize

$$\pi_2(q_1, q_L) = (600 - q_1 - q_L)q_L - 30q_L$$

First order condition:

$$600 - 2q_L - q_1 - 30 = 0$$

which yields the best response function:

$$q_L^*(q_1) = 285 - \frac{q_1}{2}$$ (BR$_L$)

Deriving Firm 1’s best response

• For any given $q_L$ and $q_H$, Firm 1 chooses $q_1$ to maximize

$$\theta[(600 - q_1 - q_H)q_1 - 60q_1] + (1 - \theta)[(600 - q_1 - q_L)q_1 - 60q_1]$$

First order condition:

$$\theta(600 - 2q_1 - q_H - 60) + (1 - \theta)(600 - 2q_1 - q_L - 60) = 0$$

which yields the best response function:

$$q_1^* = 270 - \frac{1}{2}(\theta q_H + (1 - \theta)q_L)$$
Bayesian Equilibrium quantities $q_1^*, q_H^*$ and $q_L^*$ solve

\[ q_H^*(q_1) = 255 - \frac{q_1}{2} \]

\[ q_L^*(q_1) = 285 - \frac{q_1}{2} \]

\[ q_1^* = 270 - \frac{1}{2} (\theta q_H + (1-\theta)q_L) \]

The equilibrium is

\[ q_1^* = 170 + 20\theta \]

\[ q_L^* = 200 - 10\theta \]

\[ q_H^* = 170 - 10\theta \]

As the probability $\theta$ of facing a high cost rival increases, Firm 1 becomes more aggressive in choosing its quantity. Hence both low cost and high cost rivals reduce their quantities as $\theta$ increases.
Question 3 (25 points)

Consider the following Cournot Duopoly under Asymmetric Information. Suppose that two firms (Firm 1 and Firm 2) face a high industry demand

\[ P = a - Q \]

where \( Q = q_1 + q_2 \) is the total industry output. Both firms have unit production cost \( c = \$30 \). The industry demand parameter \( a \) can either be high \( a = a_H = 600 \) with probability \( \theta \) or it can be low \( a = a_L = 300 \) with probability \( 1 - \theta \). Firm 1 knows whether demand is high or low. Firm 2 believes demand is high with probability \( \theta \) and low with probability \( 1 - \theta \). If demand is high, the two firms face

\[ P = 600 - Q \]

if demand is low, the two firms face

\[ P = 300 - Q \]

Let \( q_H \) denote the quantity produced by Firm 1 of type H (the firm who knows demand is high) and let \( q_L \) denote the quantity produced by Firm 1 of type L (the firm who knows demand is low). Finally let \( q_2 \) denote the quantity produced by Firm 2.

Find the Bayesian Equilibrium quantities \( q_1^*, q_H^* \) and \( q_L^* \) as a function of \( \theta \).
Deriving Firm 1 of Type H’s (high demand) best response
For any given $q_2$, Firm 1 of Type L chooses $q_L$ to maximize

$$\pi_1 (q_H, q_2) = (600 - q_H - q_2)q_H - 30q_H$$

First order condition:

$$600 - 2q_H - q_2 - 30 = 0$$

which yields the best response function:

$$q_H^*(q_2) = 285 - \frac{q_2}{2} \quad \text{(BR}_H)$$

Deriving Firm 1 of Type L’s (low demand) best response
For any given $q_2$, Firm 1 of Type L chooses $q_L$ to maximize

$$\pi_1 (q_L, q_2) = (300 - q_L - q_2)q_L - 30q_L$$

First order condition:

$$300 - 2q_L - q_2 - 30 = 0$$

which yields the best response function:

$$q_L^*(q_2) = 135 - \frac{q_2}{2} \quad \text{(BR}_L)$$

Deriving Firm 2’s best response For any given $q_L$ and $q_H$, Firm 2 chooses $q_2$ to maximize

$$\theta[(600 - q_H - q_2)q_2 - 30q_2] + (1 - \theta)[(300 - q_L - q_2)q_2 - 30q_2]$$

First order condition:

$$2q_2^* = \theta(570 - q_H) + (1 - \theta)(270 - q_L)$$

Bayesian Equilibrium quantities $q_2^*$, $q_H^*$ and $q_L^*$ solve

$$q_H^* = 285 - \frac{q_2^*}{2}$$

$$q_L^* = 135 - \frac{q_2^*}{2}$$

$$2q_2^* = \theta(570 - q_H^*) + (1 - \theta)(270 - q_L^*)$$
\[ 2q_2^* = \theta(570 - \left(285 - \frac{q_2^*}{2}\right)) + (1 - \theta) \left(270 - \left(135 - \frac{q_2^*}{2}\right)\right) \]

\[ 2q_2^* = \theta(570 - 285 + \frac{q_2^*}{2}) + (1 - \theta) \left(270 - 135 + \frac{q_2^*}{2}\right) \]

\[ \frac{3}{2}q_2^* = 285\theta + (1 - \theta)(135) \]

\[ \frac{3}{2}q_2^* = 150\theta + 135 \]

Therefore the full equilibrium is

\[ q_2^* = 90 + 100\theta \]

\[ q_H^* = 240 - 50\theta \]

\[ q_L^* = 90 - 50\theta \]

Note that as the probability \( \theta \) of high demand, Firm 2 (who does not know what demand it faces) becomes more aggressive. As a response, both \( q_H^* \) and \( q_L^* \) are decreasing in \( \theta \).
**Question 4 (25 points):** Player 1 (row player) and Player 2 (column player) will play one of the following two simultaneous move games. 

Player 1 knows perfectly which game is to be played. 

Player 2 believes that they are playing Game 1 with probability $\frac{1}{2}$ and Game 2 with probability $\frac{1}{2}$.

- **GAME 1 (with prob 1/2)**
  - L | R
    - T | (1,1) (0,0)
    - B | (0,0) (0,0)

- **GAME 2 (with prob 1/2)**
  - L | R
    - T | (0,0) (0,0)
    - B | (0,0) (2,2)

Denote the Player 1 type who knows Game 1 is played as Player 1 of Type 1. Denote the Player 1 type who knows Game 2 is played as Player 1 of Type 2.

Find all pure strategy Bayesian Equilibria of the above Static Bayesian game.
Pooling equilibrium #1: Type 1 and Type 2 both play T

- If Type 1 and Type 2 both play T, then if (P2) plays L, he gets
  \[
  \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2}
  \]
  if (P2) plays R, he gets
  \[
  \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0
  \]
  Hence, (P2) plays L if Type 1 and Type 2 both play T.

- Is Type 1 best responding to L
  \[
  BR_{Type1}(L) = T
  \]
  If (P2) plays L, Type 1 plays T.

- Is Type 2 best responding to L?
  \[
  BR_{Type2}(L) = T \text{ and } B
  \]
  If (P2) plays L, Type 1 plays T or B.

- Conclusion. Yes there is a pooling equilibrium in which Type 1 and Type 2 both play T and Player 2 plays L.
Pooling equilibrium #2: Type 1 and Type 2 both play B

- If Type 1 and Type 2 both play B, then if (P2) plays L, he gets
  \[
  \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0
  \]
  if (P2) plays R, he gets
  \[
  \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 2 = 1
  \]
  Hence, (P2) plays R if Type 1 and Type 2 both play B.

- Is Type 1 best responding to R?
  \[BR_{Type 1}(R) = T \text{ and } B\]

- Is Type 2 best responding to R?
  \[BR_{Type 2}(R) = B\]

- Conclusion. Yes there is a pooling equilibrium in which Type 1 and Type 2 both play B and Player 2 plays R.
Separating equilibrium #1: Type 1 plays T and Type 2 plays B

• If Type 1 plays T and Type 2 plays B, then if (P2) plays L, he gets

\[
\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2}
\]

If (P2) plays R, he gets

\[
\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 2 = 1
\]

Hence, (P2) plays R.

• Is Type 1 best responding to R by playing T?

\[BR_{Type 1}(R) = T \text{ and } B\]

So yes, Type 1 is best responding to R by playing T.

• Is Type 2 best responding to R by playing B?

\[BR_{Type 2}(R) = B\]

So yes, Type 2 is best responding to R by playing B.

• Conclusion. Yes there is a separating equilibrium in which Type 1 plays T, Type 2 plays B and Player 2 plays R.
Separating equilibrium #2: Type 1 plays B and Type 2 plays T

- If Type 1 plays B and Type 2 plays T, then if (P2) plays L, he gets

$$\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$$

If (P2) plays R, he gets

$$\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$$

Hence, (P2) is indifferent between L and R. Suppose he plays R

- Suppose (P2) plays R. Is Type 1 best responding to R by playing B?

$$BR_{Type\ 1}(R) = T \text{ and } B$$

So yes, Type 1 is best responding to R by playing B.

- Is Type 2 best responding to R by playing T?

$$BR_{Type\ 2}(R) = B$$

So NO, Type 2 is NOT best responding to R by playing T. So it is not an equilibrium when (P2) Plays R, Type 1 plays B and Type 2 plays T.

- Suppose (P2) plays L. Is Type 1 best responding to L by playing B?

$$BR_{Type\ 1}(L) = T$$

So NO, Type 1 is NOT best responding to L by playing B. So it is not an equilibrium when (P2) Plays L, Type 1 plays B and Type 2 plays T.

- Conclusion. There is No separating equilibrium in which Type 1 plays B and Type 2 plays T.