This homework assignment will only be accepted if the answers are provided on the space following each question. Treat this assignment like an exam. Write your answers only on the space provided following each question. Do not use separate sheets. Do not write your answers on other sheets of paper. For full credit, please be concise and tidy. If your answer is illegible and not well organized, you will lose points!

Question 1 (40 points): Consider the following Prisoners’ Dilemma

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1,1</td>
<td>3,0</td>
</tr>
<tr>
<td>C</td>
<td>0,3</td>
<td>2,2</td>
</tr>
</tbody>
</table>

Suppose the above stage game played infinitely. Each player has a discount factor \( \delta \in (0, 1) \). Consider the following grim-trigger strategy

Play C in the first period.

Continue to play C as long as everyone has always played C.

If a player plays D at any stage, play D forever.

Under what conditions on \( \delta \in (0, 1) \) is the above grim-trigger strategy a SPE? When is cooperation sustained?
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Question 2 (60 points) Consider two following Cournot competition between two firms, Firm 1 and Firm 2. The firms face an inverse demand function

\[ P = 600 - Q \]

where \( Q = q_1 + q_2 \) is the total output. Each unit produced costs \( c = \$60 \). Therefore the profit of each farmer is given by

\[
\pi_1 (q_1, q_2) = (600 - q_1 - q_2)q_1 - 60q_1
\]

\[
\pi_2 (q_1, q_2) = (600 - q_1 - q_2)q_2 - 60q_2
\]

Each firm \( i \) simultaneously chooses own \( q_i \) to maximize own profits \( \pi_i \).

a) (15 points) Find the Cournot NE quantities \( q_1^C \) and \( q_2^C \). Find the Cournot NE profits of the two firms, \( \pi_1^C \) and \( \pi_2^C \).
b) (15 points) Find the monopoly output level $q^M$ that maximizes the joint profits of the two firms. Find the monopoly profits.
c) (30 points) Now suppose the two firms play this Cournot game infinitely many times. Suppose each firm has a discount factor $\delta \in (0, 1)$. Consider the following grim-trigger strategy

Produce $\frac{q^m}{2}$ in the first period.

Continue to produce $\frac{q^m}{2}$ as long as everyone has always produced $\frac{q^m}{2}$.

If a player ever produces a quantity different than $\frac{q^m}{2}$, produce $q^C_i$ forever.

Under what conditions on $\delta \in (0, 1)$ is the above grim-trigger strategy a SPE? When is collusive outcome $\frac{q^m}{2}$ sustained?

Hint: You will need to find the best one-shot deviation and the resulting deviation profit for a firm who wants to deviate from the above grim-trigger strategy.