Question 1 (40 points): Consider a monopolist firm in a product market. The firm faces an inverse demand function
\[ P = 600 - 9q \]  
(1)
where \( q \) is the firm’s output and \( P \) is the price. The firm only uses labor as an input. To produce one unit of output, the firm needs \( \frac{3}{\alpha} \) units of labor. Hence, we have
\[ q(L) = \frac{\alpha L}{3} \]  
(2)
where \( L \) is the total labor input. Note that \( \alpha > 0 \) captures a worker’s productivity. Each unit of labor costs the firm a wage \( w \). The firm’s profit function is thus given by
\[ \pi (q, w) = (600 - 9q)q - wL \]  
(3)
\[ \Rightarrow \pi (L, w) = 200\alpha L - \alpha^2 L^2 - wL \]

While the Firm has exclusive control over how much labor to hire, there is a Union that has exclusive control over the wage \( w \). In the first stage of the game the Union chooses \( w \) to maximize
\[ U (w, L) = wL \]  
(4)
In the second stage, after observing \( w \) set by the Union, the Firm chooses \( L \) to maximize profits given by \( \pi (L, w) \) above in (3).
(a) (20 points) Find the Firm’s best response function $L^*(w)$ in the second stage after the Union sets a wage $w$ by maximizing the Firm’s profit function

$$\pi(L, w) = 200\alpha L - \alpha^2 L^2 - wL$$
(b) (20 points) Now solve for the optimal wage $w^*$ set by the Union in the first stage by maximizing $U(w, L) = wL$. Report the backward induction equilibrium pair $(w^*, L^*)$. 
Question 2 (40 points) Two neighbors are planning to clean their street on a Sunday. We denote the amount of time contributed by person $i$ by $c_i$ and assume that the payoff function of each individual is given by

$$u_1(c_1, c_2) = 100c_1 - 10c_1c_2 - 10c_1^2$$

$$u_2(c_1, c_2) = 100c_2 - 10c_1c_2 - 10c_2^2$$

a) (20 points) Now suppose that first Person 1 moves and chooses $c_1$ to maximize $u_1(c_1, c_2)$. In the second stage, after observing $c_1$ Person 2 chooses $c_2$. Find the backward induction equilibrium pair $(c_1^*, c_2^*)$ of this game.
b) (20 points) Now suppose the two players choose their actions $c_1$ and $c_2$ simultaneously. Find the Nash equilibrium pair $(c_1^{NE}, c_2^{NE})$ and compare it with the backward induction equilibrium pair $(c_1^*, c_2^*)$ you found in part (a). How does the ability to move first change Person 1’s equilibrium behavior? Does he contribute less or more compared to the simultaneous move case? Why?
Question 3 (20 points).

Consider the above Centipede Game. At the first stage, player 1 decides to Stop (S) or Continue (C). If player 1 plays S, the game ends. If player 1 plays C, it is then player 2’s turn to choose whether to stop or continue. When a player stops, the game ends. Otherwise the game continues with the other player choosing to stop or continue. At the very last stage, it is Player 2’s turn and his only possible action is to stop.

Describe the subgame perfect equilibrium (SPE) of this game. What is the SPE outcome?