Question 1 (40 points): Consider the following Prisoners’ Dilemma

\[
\begin{array}{cc}
D & C \\
D & 1,1 & 3,0 \\
C & 0,3 & 2,2 \\
\end{array}
\]

Suppose the above stage game played infinitely. Each player has a discount factor \( \delta \in (0, 1) \). Consider the following grim-trigger strategy

- Play C in the first period.
- Continue to play C as long as everyone has always played C.
- If a player plays D at any stage, play D forever.

Under what conditions on \( \delta \in (0, 1) \) is the above grim-trigger strategy a SPE? When is cooperation sustained?
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Question 2 (60 points) Consider two following Cournot competition between two firms, Firm 1 and Firm 2. The firms face an inverse demand function

\[ P = 600 - Q \]

where \( Q = q_1 + q_2 \) is the total output. Each unit produced costs \( c = $60 \). Therefore the profit of each farmer is given by

\[ \pi_1 (q_1, q_2) = (600 - q_1 - q_2)q_1 - 60q_1 \]

\[ \pi_2 (q_1, q_2) = (600 - q_1 - q_2)q_2 - 60q_2 \]

Each firm \( i \) simultaneously chooses own \( q_i \) to maximize own profits \( \pi_i \).

a) (15 points) Find the Cournot NE quantities \( q^C_1 \) and \( q^C_2 \). Find the Cournot NE profits of the two firms, \( \pi^C_1 \) and \( \pi^C_2 \).
b) (15 points) Find the monopoly output level $q^M$ that maximizes the joint profits of the two firms. Find the monopoly profits.
c) (30 points) Now suppose the two firms play this Cournot game infinitely many times. Suppose each firm has a discount factor $\delta \in (0, 1)$. Consider the following grim-trigger strategy:

Produce $\frac{q^M}{2}$ in the first period.

Continue to produce $\frac{q^M}{2}$ as long as everyone has always produced $\frac{q^M}{2}$.

If a player ever produces a quantity different than $\frac{q^M}{2}$, produce $q^C_i$ forever.

Under what conditions on $\delta \in (0, 1)$ is the above grim-trigger strategy a SPE? When is collusive outcome $\frac{q^M}{2}$ sustained?

Hint: You will need to find the best one-shot deviation and the resulting deviation profit for a firms who wants to deviate from the above grim-trigger strategy.