ECO 5341 (Section 1)
Spring 2016
Homework 2
Due February 9th Tuesday
Total Points: 100

This homework assignment will only be accepted if the answers are provided on the space following each question. Treat this assignment like an exam. Write your answers only on the space provided following each question. Do not use separate sheets. Do not write your answers on other sheets of paper. For full credit, please be concise and tidy. If your answer is illegible and not well organized, you will lose points!

**Question 1 (20 points):** Each of two people has one unit of resource. Each person chooses how much of the resource to use in fighting the other individual and how much to use productively. If each person $i$ devotes $y_i$ to fighting, then the total amount produced is $2 - y_1 - y_2$, and person $i$ obtains the fraction $p_i(y_1, y_2)$ of the output where

$$p_i(y_1, y_2) = \begin{cases} 
1 & \text{if } y_i > y_j, \\
1/2 & \text{if } y_i = y_j, \\
0 & \text{if } y_i < y_j.
\end{cases}$$

In words, if a player devotes more resources to fighting than the other one, she secures all the output. If both players devote the same amount of resources to fighting, they share the output equally. Each player cares only about the amount of output she receives and prefers to receive as much as possible.
a) (5 points) Is there any Nash Equilibria with \( y_1 \neq y_2 \), that is, an equilibrium in which one player allocates less resources to fighting than the other player? Explain.
b) (5 points) Is there any Nash Equilibria with $y_1 = y_2 < 1$, that is, an equilibrium in which the two players do not use all their resources to fight each other? Explain
c) (10 points) Is $y_1 = y_2 = 1$ a Nash Equilibrium? Explain.
**Question 2 (25 points)** Two neighbors are planning to clean their street on a Sunday. We denote the amount of time contributed by person $i$ by $c_i$ and assume that the payoff function of each individual is given by

$$u_1(c_1, c_2) = 100c_1 - 10c_1c_2 - 10c_1^2$$

$$u_2(c_1, c_2) = 50c_2 - 10c_1c_2 - 20c_2^2$$

a) (10 points) Find the best response function of each player. How does the best response of each player depend on the amount of time contributed by the other player to clean the street?
(b) (15 points) Find Nash Equilibrium pair of contributions \((c_1^*, c_2^*)\) of this game.
**Question 3 (30 points).** Two people can perform a task if, and only if, they both work. The cost of effort is $0 < c < 1$; and if the task is performed their payoff is 1 each. This results in the following bimatrix representation, where W stands for working, and S stands for shirking.

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0,0</td>
<td>0, $-c$</td>
</tr>
<tr>
<td>W</td>
<td>$-c,0$</td>
<td>1 $- c, 1 - c$</td>
</tr>
</tbody>
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a) (15 points) Find all the pure strategy Nash equilibria of this game.
(b) (15 points) Find all the mixed strategy Nash equilibria of this game. How does the mixed strategy equilibrium change as $c$ increases?
Question 4 (25 points)

Find all the pure and mixed strategy equilibria of the following Hawk and Dove game by constructing the best response correspondences of the players:

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0,0</td>
<td>6,1</td>
</tr>
<tr>
<td>D</td>
<td>1,6</td>
<td>3,3</td>
</tr>
</tbody>
</table>