VALUE at Risk (VaR)

- Value at Risk is an attempt to provide a single number to summarize the total risk in a portfolio of assets.
- Value at Risk is aimed at making a statement of the following form: We are $X$ percent sure that we will not lose more than $V$ dollars in the next $K$ days. The variable $V$ is the VaR of the portfolio. It is a function of two parameters: $K$ (time horizon) and $X$ (confidence level).
- Let’s denote the daily volatility of an asset (which is equal to the standard deviation of the asset’s daily return) by $\sigma_{\text{day}}$.

**Example 1:** (VaR Calculation): Consider a portfolio consisting of a position worth $10$ million in shares in IBM. Suppose $K = 10$ and $X = 99$, so that we are interested in a 99% confidence level for losses over 10 days. Assume that the volatility of IBM is 2% per day.

First let us find the standard deviation of the change in value in a 10 day period. This is given by

$$10 \text{ million} \times \sigma_{\text{day}} \times \sqrt{K} = 10 \text{ million} \times 2\% \times \sqrt{10} = 632,456.$$  

It is customary in VaR calculations to assume that the expected change in the price of a market variable is zero. With this assumption, the change in the value of a portfolio of IBM shares (worth $10$ million) over a 10-day period has a standard deviation of $632,456$ and a mean of zero. We assume that this change is normally distributed. From the table for normal distribution, one can find

$$N(-2.33) = 0.01$$

This means that there is 1% probability that a normally distributed variable will decrease in value by more than 2.33 standard deviations. Equivalently, it means that we are 99% certain that a normally distributed variable will not decrease in value by more than 2.33 standard deviations.
deviations. The 10-Day 99% VaR for our portfolio consisting of a $10 million position in IBM is therefore

$$VaR_{IBM} = 2.33 \times $632,456 = $1,473,621.$$  

**Example 2:** (VaR Calculation): Consider now a portfolio consisting of a position worth $5 million in shares in ATT. Suppose $K = 10$ and $X = 99$, so that we are interested in a 99% confidence level for losses over 10 days. Assume that the volatility of ATT is 1% per day. The 10-Day 99% VaR for our portfolio consisting of a $5 million position in ATT is therefore

$$VaR_{ATT} = 2.33 \times \left( $5,000,000 \times 0.01 \times \sqrt{10} \right) = $368,405.$$  

**Example 3:** (A Two-Asset Portfolio): Consider now a portfolio consisting of a position worth $5 million in shares in ATT and a position worth $10 million in shares in IBM. Suppose that the returns on the two shares have a bivariate normal distribution with a correlation of $\rho = 0.7$. A standard result in statistics tell us that the standard deviation of the joint portfolio is

$$\sigma_{ATT,IBM} = \sqrt{\sigma_{ATT}^2 + \sigma_{IBM}^2 + 2\rho\sigma_{ATT}\sigma_{IBM}}$$

The 10 day standard deviations of the two positions are

$$\sigma_{IBM} = $10,000,000 \times 2% \times \sqrt{10} = $632,456.$$  

$$\sigma_{ATT} = $5,000,000 \times 1% \times \sqrt{10} = $158,114.$$  

Therefore, the standard deviation of the change in the value of the portfolio consisting of both stocks over a 10-day period is given by

$$\sigma_{ATT,IBM} = \sqrt{(632,456)^2 + (158,114)^2 + 2(0.7) (632,456) (158,114)} = 751,665.$$  

which means the 10-day 99% VaR for the portfolio is

$$751,665 \times 2.33 = $1,751,379.$$