Delta Management

- The delta of a security position or portfolio measures the degree of bullishness of that position. Formally

\[
\text{portfolio delta} = \frac{\text{change in portfolio value}}{\text{change in the price of the underlying asset}}
\]

- For example, if a call option on IBM stock has delta 0.7, this means that if the IBM stock price goes up by $1, the value of the call option will go up by 70 cents.

- Delta ranges between -1 and 1. Each share of the underlying asset, by definition, is one delta point. A long position in a call option has positive delta whereas a long position in a put option has negative delta.

- By adjusting the delta, a portfolio manager can manage the exposure to movements in the price of the underlying asset.

An example of Delta Management: This example shows how a portfolio manager can use options to temporarily alter the risk exposure in a position, generate income in the process and restore the portfolio to the original state.

- Initial Conditions: Suppose on Jan 26, 2006, a portfolio manager holds 10,000 shares of IBM, currently selling at $S_0 = 33$ per share. The manager becomes slightly less optimistic and decides to reduce her bullishness by 10%. Viewing the initial 10000 share block as 100% bullish, there are several ways to reduce the bullishness by 10%. At first, the position delta is 10,000. The objective is to reduce the position delta to 9000. There are three choices:

  - Sell 1000 shares of IBM and hold cash.
  - Buy put options on IBM
  - Write (sell) call options on IBM.
• Suppose the manager chooses to write call options on IBM. In particular, suppose that a call option on IBM with strike 35 and with expiration in March sells for $2 and has delta 0.441. Each call contract gives the right to buy 100 shares of IBM at strike price 35. Accordingly, the manager has to pick number of call contracts $N$ such that

\[
10000 - (N \times 0.441 \times 100) = 9000 \Rightarrow N = 23
\]

which brings an income of $23 \times 2 \times 1000 = $4600.

• **One week later:** Suppose after one week, the IBM stock sells for $32.5 and MAR 35 calls now have a delta of 0.404 and they sell at $1.63. Therefore, the position delta has now become

\[
10000 - (23 \times 0.404 \times 100) = 9071.
\]

Suppose now that the manager decides to further reduce bullishness to 50%, worried that the price of IBM will go down further. The manager can sell more MAR 35 call contracts on IBM. Note that the manager can write 77 more call contracts (ending up in a short position in a total of 100 call contracts) and still have the short call position covered with the 10000 IBM shares (why?). This would reduce the position delta to

\[
10000 - (100 \times 0.404 \times 100) = 5960
\]

which is still higher than the target delta of 5000. Writing additional calls would leave them uncovered. Therefore, to further reduce the position delta to 5000, the manager should buy put options. Suppose APR 30 put options on IBM sell for $1.95 and have a delta of -0.310. The manager has to choose the number of put contracts $N$ to buy such that

\[
10000 - (100 \times 0.404 \times 100) - (N \times 0.310 \times 100) = 5000
\]

WHICH IMPLIES that the manager must buy 31 APR 30 put options.

• **At March expiration:** Suppose at March expiration, IBM stock price is again $33, i.e., the manager's fears did not materialize. The call options that the manager sold expire without exercise. Suppose now the manager wants to go back to the initial bullishness of 100%. All
she has to do is to sell the 31 APR 30 put options that she bought. Suppose the APR 30 put options now trade at $0.96. Therefore, selling the put position generates $2976 and takes the position delta back to 10000.

- Note that the manager generated
  - 23 call contracts sold on Jan 26 generated $4600.
  - 77 call contracts sold a week later generated $12,513.
  - 31 put contracts bought a week later generated −$6045
  - 31 put contracts sold at March expiration generated $2976

which implies that the total income from options is $14,044.

- Exercise 1: Suppose it is February 10, 2006 and you have a portfolio of 1000 shares of a stock and the current stock price is $40. You want to reduce your position delta by 40%. Suppose call option contracts on the stock with March expiration and strike price $38 have a delta of 0.480 and they trade at $2. How many of these call contracts should you buy or sell to reduce the position delta by 40%?

- Exercise 2: Suppose you have a portfolio of 400 shares of a stock. If you buy 20 call contracts on that stock with delta 0.35 and sell 15 put option contracts on the same stock with delta -0.28, what is your final position delta? Note that each call (put) contract gives the right to buy (sell) 100 shares of the stock.

- Exercise 3: Suppose you have a portfolio of 10,000 shares of some stock. You want to increase your position delta by 20%. Suppose put option contracts on the stock with expiration in 3 months have a delta of -0.480 and they trade at $2. How many of these put contracts should you buy or sell to increase the position delta by 20%?
a) What is the value of the call option at 75, if the holder shouts at 60?

\[
90\Delta - 45 = 50\Delta - 5 \Rightarrow \Delta = 1
\]

\[
50\Delta - 5 = 50(1) - 5 = 45
\]

\[
\frac{45}{1 + 0} = 45 = 75(\Delta) - c
\]

\[c = 30.\]

b) What is the value of the call option at 40, if the holder shouts at 60?

which implies that value of the call at 40 (if the holder shouts at 60) is 5.
c) What is the value of the call option at 60, if the holder shouts at 60? Given the answers to a and b, if the holder shouts at 60, we have

\[
\begin{align*}
75\Delta - 30 &= 40\Delta - 5 \Rightarrow \Delta = 0.714 \\
40\Delta - 5 &= 40(0.714) - 5 = 23.56 \\
\frac{23.56}{1+0} &= 23.56 = 60(\Delta) - c \\
c &= 19.28.
\end{align*}
\]

- d) What is the value of the call option at 75, if the holder DOES NOT shout at 60?

This means that the holder will shout at 75, which implies

\[
\begin{align*}
90\Delta - 45 &= 50\Delta - 20 \Rightarrow \Delta = 0.625 \\
50\Delta - 20 &= 50(0.625) - 20 = 11.25 \\
\frac{11.25}{1+0} &= 11.25 = 75(0.625) - c \\
c &= 35.625.
\end{align*}
\]

e) What is the value of the call option at 40, if the holder DOES NOT shout at 60?

Note that at 40, the value of the call is zero, whether the holder shouts or not.
f) What is the value of the call option at 60, if the holder DOES NOT shout at 60?

Given the above answers, if the holder DOES NOT shout at 60, we have

\[
\begin{array}{c}
60 \\
\uparrow \\
75 \\
\quad (35.625) \\
\downarrow \\
40 \\
\quad (0)
\end{array}
\]

Solving this tree yields

\[
75\Delta - 35.625 = 40\Delta \Rightarrow \Delta = 1.01
\]

\[
40\Delta = 40(1.01) = 40.4
\]

\[
\frac{40.4}{1 + 0} = 40.4 = 60(1.01) - c
\]

\[
c = 20.2
\]

which implies that NOT SHOUTING at 60 (waiting to shout at 75) is better.