Forward Contracts

• A forward contract is a particularly simple derivative. **It is an agreement to buy or sell an asset at a certain future time at a certain price.**

• It can be contrasted with a **spot contract**, which is an agreement to buy or sell an asset today.

• A forward contract is traded in the **over-the-counter market**, usually between two financial institutions or between a financial institution and one of its clients.

• One of the parties to a forward contract assumes a **long position** and **agrees to buy** the underlying asset on a certain specified future date for a certain specified price. The other party assumes a **short position** and **agrees to sell** the underlying asset on the same for the same price.

• The price in a forward contract is known as the **delivery price**.

• Forward contracts are commonly used to hedge foreign currency risk.

**Example 1:** (Hedging Currency Risk with a Forward Contract)

• Suppose it is April 5 of a certain year and the treasurer of a U.S. corporation knows that the corporation will receive 1 million EUROs in three months (on July 5th), and wants to hedge against the exchange rate moves. The treasurer could contact a bank, find out that the exchange rate for a 3-month forward contract on EURO is $1.25, and agree to sell 1 million EUROs. In this case, the corporation takes a short forward position (agrees to sell), whereas the bank assumes a long forward position (agrees to buy). This forward contract eliminates all exchange rate risk, since the corporation will receive $1.25 million no matter what happens to the Euro currency rate in the course of the next 3 months.
Forward Prices and Arbitrage

Let us now illustrate how the spot prices are linked to forward prices by a no-arbitrage argument.

- Arbitrage involves locking in a profit by simultaneously entering into transactions in two or more markets to exploit a pricing anomaly. Such opportunities to make riskless profits are quite rare, since when the traders start moving to exploit them the prices adjust accordingly so that the arbitrage opportunity disappears.

Example 2 (Arbitrage with a Forward Contract on Gold).

This example illustrates the possibility of arbitrage when the delivery price on a forward contract is too high or too low. Consider a trader who owns one ounce of gold today. Suppose the spot prices for gold is such that in the spot market traders can buy an ounce of gold at $P_B = $100 and sell at $P_S = $95. Furthermore, suppose that the 1-year borrowing and lending rates are such that traders can borrow at $R_B = 5\%$ and lend at $R_L = 4\%$ a year.

- Suppose first that the delivery price for a one year forward contract on gold is $F = $107. In this case, the following arbitrage strategy yields a riskless profit.
  
  - * Borrow $100 at 5\%, buy one ounce of gold in the spot market and take a short position in the forward contract.
  - * To see why, note that at time of delivery for the forward, the trader will sell the gold she bought for $107, whereas she has to pay back the loan at $100(1+0.05) = $105, which leaves her with $107 - $105 = $2 profits.
  - * Note that once traders start exploiting this arbitrage opportunity by taking short forward positions, there will be an excess supply to deliver gold at $107, which will drive the 1-year gold forward delivery price down.
  - * For the arbitrage opportunity to disappear, the delivery price $F$ should be less than

\[
F < P_B(1 + R_B) = 100(1 + 0.05) = 105
\]

\[
\implies F < 105.
\]
Example 2 (continued)

- Suppose now that the delivery price for a one year forward contract on gold is $F = 96$. In this case, the following arbitrage strategy yields a riskless profit (recall that our trader own one ounce of gold to begin with).
  - * Sell the gold today at $P_S = 95$, lend the proceeds at $R_L = 4\%$, and take a LONG position in the forward contract.
  - To see why, note that at time of delivery for the forward contract, the trader will BUY the gold at $F = 96$, whereas she will receive $95(1 + 0.04) = 98.8$ (for the funds she invested at 4%), which leaves her with $98.8 - 96 = 2.8$ profits.
  - Note that once traders start exploiting this arbitrage opportunity by taking long forward positions, there will be an excess demand to be delivered gold at $96$, which will drive the 1-year gold forward delivery price up.
  - For the arbitrage opportunity to disappear, the delivery price $F$ should be more than

$$F > P_S(1 + R_L) = 95(1 + 0.04) = 98.8 \implies F > 98.8$$

Practice Problems

- **Problem 1.** A U.S. company expects to pay 1 million Euros in six months. How can they use forward contracts to hedge against the exchange rate risk?

- **Problem 2.** The price of gold is currently $500$ per ounce. The forward price for delivery in one year is $700$. An arbitrage trader can borrow money at 10\% per annum. Identify an arbitrage strategy.

- **Problem 3.** A trader owns one unit of gold. The trader can buy gold at $50$ per ounce and sell it at $40$ per ounce in the spot market. She can borrow money at 6\% per year and can invest money at 5\% per year. For what range of one-year gold forward price $F$ does this trader have no arbitrage opportunities?