Question 1 (15 points)

A trader currently holds 500 shares of IBM stock. The trader also has $20,000 in cash. Consider the following two strategies that the trader can follow.

Strategy 1: The trader holds the 500 shares for one year, and invests $20,000 cash in a risk free bond for an annual return of 10%.

Strategy 2: The trader buys 500 put options on IBM with strike price X=60 that expire in one year. The price of each option is \( p = $10 \). The trader then holds 500 IBM shares and invests the remaining cash in the risk free bond for an annual return of 10%.

a) (7 points) Suppose IBM share price at the expiration date is \( S_T = 55 \). Which one of the two strategies, Strategy 1 or Strategy 2, yields a higher payoff to the trader?

\[
\begin{align*}
\text{Strategy 1 yields} & \quad 500(S_T) + 20,000(1.10) \\
& = 500(55) + 22,000 \\
& = \$49,500
\end{align*}
\]

\[
\begin{align*}
\text{Strategy 2 yields} & \quad 500(60) + 15,000(1.10) \\
& = \$46,500
\end{align*}
\]

\[ A+ S_T = 55 \quad \Rightarrow \quad \text{Strategy 1 yields higher payoff} \]
b) (8 points) For what values of IBM share price $S_T$ at the expiration date, does Strategy 2 prove to be the better one?

<table>
<thead>
<tr>
<th></th>
<th>$S_T &lt; 60$</th>
<th>$S_T &gt; 60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRATEGY 1</td>
<td>$500 S_T + 20,000 (1.10)$</td>
<td>$500 S_T + 20,000 (1.10)$</td>
</tr>
<tr>
<td>STRATEGY 2</td>
<td>$500 (60) + 15,000 (1.10)$</td>
<td>$500 S_T + 15,000 (1.10)$</td>
</tr>
</tbody>
</table>

STRATEGY 2 is better if and only if:

$500 (60) + 15,000 (1.10) > 500 S_T + (20,000)(1.10)$

$30,000 + 16,500 > 500 S_T + 22,000$

$S_T < \frac{24,500}{500}$

$S_T < 49$
Question 2 (Duration Matching) (15 points)

Consider a bank with the following balance sheet:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Value</th>
<th>Duration of the Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>4yr loan @5%</td>
<td>$5,000</td>
<td>3</td>
</tr>
<tr>
<td>5yr loan @5%</td>
<td>$5,000</td>
<td>4</td>
</tr>
<tr>
<td>6yr loan @6%</td>
<td>$5,000</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Value</th>
<th>Duration of the Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4yr loan @ Libor</td>
<td>$5,000</td>
<td>1</td>
</tr>
<tr>
<td>3yr loan @5%</td>
<td>$5,000</td>
<td>2</td>
</tr>
<tr>
<td>6yr loan @6%</td>
<td>$10,000</td>
<td>4.5</td>
</tr>
</tbody>
</table>

a) (7 points) Find the duration gap. When does the net worth suffer a loss, as interest rates go up or as interest rates go down?

\[
\text{Duration of Assets} = D_A = \left( \frac{5000}{15000} \right) 3 + \left( \frac{5000}{15000} \right) 4 + \left( \frac{5000}{15000} \right) 5
\]

\[
\Rightarrow D_A = 4 \quad V_A = 15,000
\]

\[
\text{Duration of Liabilities} = D_L = \left( \frac{5000}{20000} \right) 1 + \left( \frac{5000}{20000} \right) 2 + \left( \frac{10000}{20000} \right) 4.5
\]

\[
\Rightarrow D_L = (0.25) 1 + (0.25) 2 + (0.5) 4.5
\]

\[
D_L = 3 \quad V_L = 20,000
\]

\[
D_{gap} = \frac{D_A V_A - D_L V_L}{V_A} = \frac{4(15000) - 3(20000)}{20,000} = 0
\]

\[
\Rightarrow D_{gap} = 0 \quad \Rightarrow \text{Net worth is immune to interest rate risk. It does not suffer any loss as interest rates go up or down.}
\]
b) (8 points)

- Suppose the portfolio manager wants to make Duration Gap equal to +0.40.
- For that purpose, the manager wants to swap $x$ of the 6yr loan @ 6% with a 6yr loan at Libor.
- The duration of 6yr loan at Libor is 2.

What is the swap size $x$ that makes duration gap equal to +0.40?

To make \( D_{\text{gap}} = 0.4 \) we need:

\[
D_{\text{gap}} = \frac{D_A V_A - D_L V_L}{V_A} = 0.4
\]

\[
\Rightarrow 4\left(\frac{15,000}{15,000}\right) - D_L \left(\frac{20,000}{15,000}\right) = 0.4 \Rightarrow D_L = 2.7
\]

To achieve \( D_{L_{\text{target}}} = 2.7 \) we need:

\[
D_{L_{\text{target}}} = 2.7 = \left(\frac{5,000}{20,000}\right) + \left(\frac{5000}{20,000}\right) + \left(\frac{10000-x}{20,000}\right) 4.5 + \left(\frac{x}{20,000}\right) 2
\]

\[
\Rightarrow 2.7 = \frac{5000 + 10,000 + 45000 - 4.5x + 2x}{20,000}
\]

\[
\Rightarrow 2.7 \times 20,000 = 60,000 - 2.5x
\]

\[
2.5x = 60,000 - 54000
\]

\[
\Rightarrow x = \frac{6000}{2.5} \Rightarrow x = 2400
\]
Question 3 (10 points)

Consider a trader who

- Sells 10 call option contracts on BAC with call delta \(0.60\). (each call option contract contains 100 call options)
- Sells 10 put option contracts on BAC with put delta \(-0.40\). (each put contract contains 100 put options)

How many shares of the underlying BAC stock should the trader buy or sell to reduce the position delta to zero? You need to state whether the trader should buy or sell BAC shares to make position delta zero.

\[
\text{LET'S FIRST COMPUTE POSITION DELTA}
\]

\[
\text{POSITION DELTA} = -10 (100)(0.6) - 10 (100)(-0.4)
\]

\[
= -600 + 400
\]

\[
\text{POSITION DELTA} = -200
\]

\[
\text{TO REDUCE POSITION DELTA TO ZERO, TRADER SHOULD BUY 200 SHARES OF BAC}
\]
**Question 4 (14 points):** Consider two PUT options on the same stock with the same expiration date.

- PUT Option #1 has a strike price of $X_1 = 220$ and its price is given by $p_1 = 40$.
- PUT Option #2 has a strike price of $X_2 = 200$ and its price is given by $p_2 = 15$.

The NET risk free rate of return from today until the expiration date is $r = 0\%$.

**Hint:** Recall that we must have $p_1 - p_2 < X_1 - X_2$

a) (4 points) State whether arbitrage is possible. Precisely specify the arbitrage position

\[
\begin{align*}
\text{We must have} & \quad p_1 - p_2 < X_1 - X_2 \\
& \quad p_1 - p_2 < 220 - 200 \\
& \quad p_1 - p_2 < 20 \\
\end{align*}
\]

**But we have** $p_1 - p_2 = 40 - 15 = 25 > 20$

**Therefore arbitrage is possible.**

**Arbitrage position**

- **Sell the PUT with** $X_1 = 220$ **at** $p_1 = 40$
- **Buy the PUT with** $X_2 = 200$ **at** $p_2 = 15$
- **Invest** $p_1 - p_2 = 40 - 15 = 25$ **at** $r = 0\%$
b) (10 points) Find the arbitrage profit for all possible values of \( S_T \) the stock price at the expiration date.

\[
\text{ARBITRAGE PROFIT}
\]

If \( S_T < 200 \) → \( -(0) + (0) + 25(1+0\%) = $25 \)

If \( 200 < S_T < 220 \) → \( -(220-S_T) + 0 + 25 = S_T - 195 > 0 \)

If \( S_T > 220 \) → \( -(220-S_T) + (200-S_T) + 25 = $5 \)
Short Answer questions: Total points: 16

Question 5 (4 points) State whether the following statement is true or false and explain why in one or two sentences. Be precise and concise for full credit.

Statement: Consider a bond trader who carries a bond portfolio with a positive duration gap. If the trader reduces the duration of liabilities, this will help him to protect net worth when interest rates go up.

\[ D_{\text{gap}} > 0 \]. If trader reduces \( D_L \), \( D_{\text{gap}} \) becomes more positive.

with \( D_{\text{gap}} > 0 \) as \( r \uparrow \) net worth declines

Trader must increase (not reduce) \( D_L \)

\[
\text{FALSE}
\]

Question 6 (4 points) State whether the following statement is true or false and explain why in one or two sentences. Be precise and concise for full credit.

Statement: Consider a bond trader who carries a bond portfolio with a negative duration gap. If the trader expects the interest rates to go up, he/she should increase the duration of liabilities to benefit from such an increase

\[ D_{\text{gap}} < 0 \]. If trader increases \( D_L \), duration gap will become more negative.

If \( r \uparrow \) and if \( D_{\text{gap}} < 0 \) net worth increases

\[
\text{TRUE}
\]
Question 7 (4 points) State whether the following statement is true or false and explain why in one or two sentences. Be precise and concise for full credit.

Statement: If an investor BUYS call options on a stock and SELLS put options on the same stock, then this investor can always offset his/her risk with respect to the underlying stock price by buying shares of the underlying stock.

Buying call options increases delta
Selling put options increases delta
To offset delta, TRADER MUST SELL
SHARES of underlying stock.

FALSE

Question 8 (4 points) State whether the following statement is true or false and explain why in one or two sentences. Be precise and concise for full credit.

Statement: If the duration gap is positive, increasing the duration of the asset side will further increase the duration gap and will expose the balance sheet to more losses if interest rates go up.

\[ D_{gap} > 0, \text{ If } D_{A} \uparrow \text{ duration gap will increase further and become more positive.} \]

If \( D_{gap} > 0 \) and \( r \uparrow \) net worth \( \downarrow \)

TRUE
Question 9: (Interest Rate Swap) (15 points)

A and B have been offered the following rates on a $10 million 10-year loan.

<table>
<thead>
<tr>
<th>Company</th>
<th>Fixed Rate</th>
<th>Floating Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12%</td>
<td>Libor + 9%</td>
</tr>
<tr>
<td>B</td>
<td>9% (3%)</td>
<td>Libor + 3% (6%)</td>
</tr>
</tbody>
</table>

A requires a floating rate loan. B requires a fixed rate loan.

A) (5 points) Which company has comparative advantage in Fixed Rate Loan market? Which company has comparative advantage in Floating Rate Loan market? What is size of the potential gain from a swap agreement in the above situation?

A has comp. advantage in FIXED.
B has comp. advantage in FLOATING.

Surplus = 6% - 3% = 3%
B (10 points)

- Suppose A and B CANNOT engage directly with each other and they need a BANK to act as an intermediary.

- Design a pair of swaps such that the surplus is shared equally between A, B and the Bank. (Each get 1/3 of the surplus)

**Purpose of Swaps**

1% A → ENDS up at LIBOR + 8%
1% B → ENDS up at 8%
1% Bank

12% A → LIBOR → BANK → B → Libor + 3%
4% A ← BANK ← B ← 5%
Question 10: (Portfolio Delta) (15 points)

If the following two portfolios contain the same delta points, what should be the number of CALL options contracts $N$ in portfolio $B$?

**Portfolio A:** 30 long put option contracts on IBM (each contract contains 100 put options) with put delta -0.30.

**Portfolio B:** 200 shares of IBM and $N$ short call option contracts on IBM (each contract contains 100 call options) with call delta +0.50.

\[
\text{DELTA of A} = \text{DELTA of B}
\]
\[
+ 30(100)(-0.30) = 200(1) - N(100)(0.50)
\]
\[
-900 = 200 - 50N
\]
\[
50N = 1100
\]
\[
N = 22
\]