Financial Economics 4378
FALL 2013
Practice Problems
Not to be submitted, for self-study
Solutions to be provided in class

Question 1: (Payoff From a Spread)

Consider the following spread created by call options on a stock:
A short position in a call option with a price of $5 and with a strike price of $50,
And a long position in a call option with a price of $2 and with a strike price of $60.
Both call options have the same expiration date.

a) What is the payoff to this spread if at expiration, stock price is $S_T=40$?

b) What is the payoff to this spread if at expiration, stock price is $S_T=65$?

c) What is the payoff to this spread if at the expiration date the stock price $S_T$ is between 20 and 30?

\[
\begin{align*}
\text{a) } & S_T = 40, \quad \text{payoff} = -(0) + (0) + 5 - 2 = +3 \\
\text{b) } & S_T = 65, \quad \text{payoff} = -(65 - 50) + (65 - 60) + 5 - 2 = -7 \\
\text{c) } & \text{If } 20 < S_T < 30 \Rightarrow -0 + 0 + 5 - 2 = +3
\end{align*}
\]
Question 2: (Lower Bound on Put Option Price and Arbitrage Profit)

A European put option (on a stock) that expires in a year has a strike price of $99. The current stock price is $75 and the one-year risk free interest rate is 10%. The price of this put is $10.

(a) Is arbitrage possible? What is the arbitrage position?

\[
\text{Lower bound on } P = \frac{X - S_0}{1 + r} = \frac{99 - 75}{1.10} = 15
\]

Since \( P = 10 < \text{lower bound} = 15 \), arbitrage possible

\[
\text{Arbitrage Position} = \begin{cases} 
\text{Buy the put at } P = 10 \\
\text{Buy the stock at } S_0 = 75 \\
\text{Borrow } P + S_0 = 85 \text{ at } r = 10\%
\end{cases}
\]

(b) Find the arbitrage profit for this arbitrage strategy, if at expiration the stock price turns out to be \( S_T = 90 \)?

\[
\text{Payoff at } S_T = 90 \rightarrow 99 - 85(1.10) = 5.5
\]

c) (5 points) What is the minimum arbitrage profit in this situation?

\[
\text{Payoff at any } S_T < 99 \rightarrow 99 - 85(1.10) = 5.5
\]

Minimum profit is $5.5 which happens for any \( S_T < 99 \).
Question 3 (15 pts)

A trader holds 200 shares of IBM stock. The trader also has $10,000 in cash. Consider the following two strategies that the trader can follow.

Strategy 1: The trader holds the 200 shares for one year, and invests 10,000 cash in a risk free bond for an annual return of 10%.

Strategy 2: The trader buys 200 put options on IBM with strike price X=200 that expire in one year. The price of each option is p=15. The trader then holds 200 IBM shares and invests the remaining cash in the risk free bond for an annual return of 10%.

For what values of IBM share price $S_T$ in one year at the expiration date, does Strategy 2 prove to be the better one?

**Strategy 1** \[ \text{Payoff} = 200 \cdot S_T + 10,000 \cdot (1.10) \]
\[ = 200 \cdot S_T + 11,000 \]

**Strategy 2**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T &lt; 200$</td>
<td>$200(200) + 7000(1.10)$ = $47,700$</td>
</tr>
<tr>
<td>$S_T &gt; 200$</td>
<td>$200S_T + 7000(1.10)$</td>
</tr>
</tbody>
</table>

**Strategy 2 is better if**

\[ 36,600 > 200S_T \]

\[ S_T < 183 \]
Question 4:

Consider the following portfolios:

Portfolio A: 20 short put options on IBM stock with strike price $50 and 50 shares of IBM stock. Each put gives the right to sell one share of the IBM stock.

Portfolio B: 20 long positions in call options on IBM stock with strike price $40 and \( z \) shares of IBM stock. Each call gives the right to buy one share of the IBM stock.

Suppose that, if IBM stock price at expiration date turns out to be \( S_T = \$60 \), then both portfolios have the same payoff.

What is the number of shares \( z \) in Portfolio B?

\[
\begin{array}{|c|c|}
\hline
& Value of A & \text{Value of B} \\
\hline
\text{AT } S_T = 60 & -20(0) + 50S_T = 3000 & +20(60-40) + zS_T \\
\hline
& (\text{since } S_T = 60) & = 400 + 60z \\
\hline
\end{array}
\]

\[
3000 = 400 + 60z
\]

\[
z = \frac{2600}{60} = 43.33 \text{ shares}
\]
**Question 5:**

Consider a trading position which involves

A long position in a call option with a strike price $X = 70$ and a price $c = 10$.
A short position in a put option with a strike price $X = 70$ and price $p = 10$.

Both options have the same underlying stock and the same expiration date.

Find and draw the payoff diagram for this position as a function of $S_T$

If $S_T < 70$ \[ \rightarrow + (0) - (70 - S_T) + 10 - 10 = S_T - 70 \]

If $S_T > 70$ \[ \rightarrow (S_T - 70) - 0 + 10 - 10 = S_T - 70 \]
Question 6:

A trader sells 100 shares of IBM stock short at share price $50 each and hence generates $5,000. Consider the following two strategies that the trader can follow.

Strategy 1: The trader holds the short position of 100 shares for one year, and invests $5,000 cash in a risk free bond for an annual return of 10%.

Strategy 2: The trader buys 100 call options on IBM with strike price X=70 that expire in one year. The price of each call option is p=10. The trader holds the short position of 100 shares for one year and invests the remaining cash in the risk free bond for an annual return of 10% (note that now the trader can only invest 4,000 since 1000 is spent on call options).

For what values of IBM share price $S_T$ in one year at the expiration date, does Strategy 2 prove to be the better one?

\[
\text{STRATEGY 1} \quad \rightarrow \quad 5000 \times (1.10) - 100S_T
\]

\[
\text{STRATEGY 2}
\]

<table>
<thead>
<tr>
<th>If $S_T &lt; 70$</th>
<th>$4000(1.10) - 100S_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $S_T &gt; 70$</td>
<td>$4000(1.10) - 70(100) = 4400 - 7000$</td>
</tr>
</tbody>
</table>

Strategy 2 is better if:

\[
5000 - 2600 > 5500 - 100S_T
\]

\[
100S_T > 8100
\]

\[
S_T > 81
\]