Question 1 (25 points) A company recently issued 10 year bonds at a price of $1000. These bonds pay $20 interest every 6 months. Their price remained stable since they were issued, and they still trade at par value of $1000.

Due to additional financing needs, the company wants to issue new bonds with a maturity of 5 years, a par value of $1000 and that pay $50 coupons every 6 months. If the investors require the same annual rate of return as they require from the existing bonds, what would be the value of the new bonds?

Answer: Since the old one still trade at par value, we must have

\[ r_d \text{ annual on old bonds} = \text{annual coupon rate of old bonds} \]

and hence

\[ r_d \text{ annual on old bonds} = \frac{20}{1000} \times 2 = 4\% \]

Therefore, the value of new bonds will be given by

\[
V_{new \text{ bonds}} = 50(PVIFA)_{10.2\%} + 1000(PVIF)_{10.2\%} \\
= 50(8.983) + 1000(0.82) \\
= 1269.15
\]
Question 2 (25 points) Assume that you are considering the purchase of a $1000 par value bond that pays a coupon of $90 every six months and has 10 years to go before it matures. If you buy this bond, you expect to hold it for three years and sell it in the market. You expect the market to require an annual rate of 12% when you sell the bond. What will be the value of the bond when you sell it at the end of three years?

Answer: After 3 years at $r_d$ annual= 12%, we have 14 coupons left

$$V_{bond} = 90(PVIFA)_{14.6\%} + 1000(PVIF)_{14.6\%}$$
$$= 90(9.295) + 1000(0.442)$$
$$= 1278.55$$
Question 3 (20 points) A portfolio manager has a portfolio of $200,000. Assume that the risk-free rate is \( r_{RF} = 1\% \). The current portfolio beta is \( \beta = 1.6 \) and the required return on this portfolio is \( r_p = 9\% \).

The manager invests an additional $300,000 in a portfolio with beta 1.8.

What is the new required return on the manager’s final portfolio after this additional $300,000 is invested in the new portfolio?

Answer: Let’s first find the final portfolio beta

\[
\beta_p^{final} = \left(\frac{200,000}{200,000 + 300,000}\right)1.6 + \left(\frac{300,000}{200,000 + 300,000}\right)1.8
\]
\[
= 0.4(1.6) + 0.6(1.8)
\]
\[
\beta_p^{final} = 1.72
\]

We also can find \( r_m \) from

\[
r_p = 9\% = r_{RF} + \beta(r_m - r_{RF})
\]
\[
9\% = 1\% + 1.6(r_m - 1\%)
\]
\[
\Rightarrow r_m = 6\%
\]

Hence

\[
r_p^{final} = r_{RF} + \beta_p^{final}(r_m - r_{RF})
\]
\[
= 1\% + 1.72(6\% - 1\%)
\]
\[
r_p^{final} = 9.6\%
\]
**Question 4 (25 points)** A portfolio manager has a $300,000 portfolio with a required return of 9%. Assume that the required return on the market portfolio is $r_M = 6\%$ and the risk-free rate is $r_{RF} = 1\%$.

The manager wants to invest **some additional funds** in some **new portfolio with a beta 2.8**, so that the new portfolio will have a required rate of return of 11%.

What is the dollar amount that the manager should invest in the new portfolio with $\beta_p^{new} = 2.8$ so that the final portfolio will have a required rate of return of 11%?

Let’s first find the final portfolio beta

$$r_{p^{final}} = r_{RF} + \beta_p^{final}(r_m - r_{RF})$$

11% = 1% + $\beta_p^{final}(6\% - 1\%)$

$\beta_p^{final} = 2$

Let’s now find the current portfolio beta

$$r_{p^{current}} = r_{RF} + \beta_p^{current}(r_m - r_{RF})$$

9% = 1% + $\beta_p^{current}(6\% - 1\%)$

$\beta_p^{current} = 1.6$

Hence we need

$$\beta_p^{final} = 2 = \left( \frac{300,000}{300,000 + x} \right) \beta_p^{current} + \left( \frac{x}{300,000 + x} \right) 2.8$$

$$2 = \left( \frac{300,000}{300,000 + x} \right) 1.6 + \left( \frac{x}{300,000 + x} \right) 2.8$$

2(300,000 + x) = 480,000 + 2.8x

600,000 + 2x = 480,000 + 2.8x

x = \frac{120,000}{0.8} = $150,000