Question 1:

- Consider two identical countries a and b.

- In each country, the population of young and old generation is 500 ($N_t^a = N_t^b = 500$),

- Each young person in A wants real money balances worth 100 goods.

- Each young person in B wants real money balances worth 100 goods.

- The population in both countries is constant, hence in each country there are 500 young and 500 old individuals.

Assume that the total fiat money stock of country a is $50,000 and that of country b is 50,000 pesos. There are no foreign currency controls and the fiat money stocks are equally disbursed among the initial old of both countries, i.e., each old person in the two countries hold

\[
\frac{50,000}{1,000} = 50 \quad \text{and} \quad \frac{50,000}{1,000} = 50 \text{ pesos.}
\]

regardless of citizenship. Finally assume that the exchange rate is fixed at

\[
\tilde{e} = \frac{v_a}{v_b} = 3
\]

where $v_a$ is the real value of one dollar and $v_b$ is the real value of one peso.
a) (25 points) Find the values of \( v_a \) and \( v_b \).

\[
N_t^a(100) + N_t^b(100) = v_a M_t^a + v_b M_t^b \\
500(100) + 500(100) = v_a 50,000 + v_b 50,000
\]

but since \( v_a = 3v_b \), this becomes

\[
100,000 = 200,000v_b \\
\rightarrow v_b = 0.5 \\
\rightarrow v_a = 1.5
\]

b) (25 points) Find the old age consumptions \( c_2^a \) and \( c_2^b \) in the two countries

\[
c_2^a = c_2^b = 50v_a + 50v_b = 50(1.5) + 50(0.5) = 100 \text{ goods}
\]
c) (25 points) Suppose now that every member of the initial old of both countries decides to reduce their peso holdings (country b money). Each member of the initial old in both countries will turn in 30 pesos in exchange for dollars. **Assume further that the monetary authority of country a has agreed to cooperate by printing as much of its currency (dollars) as demanded to keep the exchange rate fixed at \( \bar{e} = 3 \).**

Find the values of \( v_a \), \( v_b \), and the old age consumption levels \( c^a_2 \) and \( c^b_2 \) under this cooperative regime.

Answer: Note that the total pesos sold is given by

\[
30 \times 1,000 = 30,000
\]

At \( \bar{e} = 3 \), we must have 30,000 pesos sold for $10,000. Hence Country prints additional $10,000 and we have

\[
$50,000 + $10,000 = $60,000.
\]

Therefore we now have

\[
N^a_t(100) + N^b_t(100) = v_a M^a_t + v_b M^b_t
\]

\[
500(100) + 500(100) = v_a 60,000 + v_b 20,000
\]

\[
100,000 = (3v_b)(60,000) + v_b(20,000)
\]

\[
100,000 = (200,000) v_b
\]

\[
\rightarrow v_b = 0.5
\]

\[
\rightarrow v_a = 1.5
\]

The old age consumptions \( c^a_2 \) and \( c^b_2 \) in the two countries now become.

\[
c^a_2 = c^b_2 = 6v_a + v_b = 60(1.5) + 20(0.5) = 100 \text{ goods}.
\]
d) (25 points) Suppose again that every member of the initial old of both countries decides to cut their peso holdings (country b money). Each member of the initial old in both countries will turn in 30 pesos in exchange for dollars. **Assume now that the monetary authority of country a DOES NOT cooperate** by printing as much of its currency (dollars) as demanded, and therefore country b needs to unilaterally defend the exchange rate at \( \bar{e} = 3 \) by taxing its own old citizens.

Find the values of \( v_a, v_b, \) and the old age consumption levels \( c^a_2 \) and \( c^b_2 \) under this unilateral defense of the exchange rate regime and compare the solution with the cooperative solution.

Without A’s cooperation, we now have

\[
N_t^a(100) + N_t^b(100) = v_a M_t^a + v_b M_t^b
\]

\[
500(100) + 500(100) = v_a 50,000 + v_b 20,000
\]

\[
100,000 = (3v_b) (50,000) + v_b (20,000)
\]

\[
100,000 = (170,000) v_b
\]

\[
\Rightarrow v_b = \frac{10}{17}
\]

\[
\Rightarrow v_a = \frac{30}{17}
\]

Let’s find the tax per old of Country B

\[
\text{tax per old of B} = \frac{10,000v_a}{500} = 20v_a = \frac{20 \times 30}{17} = \frac{600}{17}
\]

The old age consumptions \( c^a_2 \) and \( c^b_2 \) in the two countries now become. The old age consumptions \( c^a_2 \) and \( c^b_2 \) in the two countries now become.

\[
c^a_2 = 60v_a + 20v_b = 60(\frac{30}{17}) + 20(\frac{10}{17}) = \frac{2000}{17} = 117
\]

\[
c^b_2 = 60v_a + 20v_b - \text{Tax} = 60(\frac{30}{17}) + 20(\frac{10}{17}) - \frac{600}{17} = \frac{1400}{17} = 83
\]