ON THE EFFICIENCY OF JOB SEARCH WITH SOCIAL NETWORKS

PIERRE CAHUC
Ecole Polytechnique

FRANÇOIS FONTAINE
Université de Strasbourg and IZA

Abstract

This paper provides a simple matching model in which unemployed workers and employers can be matched together through social networks and through more efficient, and also more costly, methods. In this framework, decentralized decisions to utilize social networks in the job search process can be inefficient and give rise to multiple equilibria. More precisely, in a decentralized equilibrium, social networks can be overutilized, with respect to an efficient allocation, in some circumstances and underutilized in others. Moreover, the existence of different job search methods can give rise to a higher job search intensity than the efficient one. This is in sharp contrast with the standard result, derived in matching models, according to which search intensity is always too low if not efficient. Finally, in the presence of different job search methods, conditional unemployment benefits hikes, which can be used as a coordination device to improve welfare when individuals.

1. Introduction

To a large extent, economic analysis conveys the idea that search intensity of unemployed workers is too low. The design of unemployment insurance
systems is indeed deeply influenced by the existence of moral hazard, which implies that unemployed workers lower their search intensity if they are not strongly urged to find a job. One strand of literature has focused on the design of optimal unemployment systems in partial equilibrium models, in which unemployed workers face job arrival rates that depend on their search intensity (Shavell and Weiss 1979, Hopenhayn and Nicolini 1997). It shows that optimal systems involve a replacement ratio that decreases throughout the unemployment spell and a wage tax after reemployment that increases with the unemployment spell. Another strand is more focused on equilibrium models of the labor market in which matches between job seekers and job vacancies are explicitly taken into account (Pissarides 2000). In such a framework, externalities yielded by the decisions of participants on both sides of the market are taken into account. This strand of literature also helps to fuel the idea that job search intensity is too low. In his synthesis, Pissarides (2000, p. 194) concludes that job search intensity is efficient only if the surpluses provided by job–worker matches are shared according to the Hosios–Pissarides rule1 (Hosios 1990, Pissarides 2000). Otherwise, search intensity is too low. It should be noted that some contributions have tried to stress that unemployment insurance may raise efficiency by improving the average productivity of job–worker matches (Diamond 1981, Acemoglu and Shimer 1999, 2000, Marimon and Zilibotti 1999). In this context, unemployment insurance allows workers to bear long unemployment spells in order to find good matches, with high productivity. However, even if the increase in matches quality may improve efficiency, job search intensity is always too low if not efficient.

In this paper, we argue that the result according to which job search intensity is too low relies on a very restrictive conception of job search activity. Indeed, the efficiency of job search intensity has been analyzed in a framework in which search effort is treated as unidimensional. However, in real life, unemployed workers can use a large range of methods to find a job. They can send direct applications, but they can also use friends and relatives, newspapers, private agencies, and state agencies. It is now well known that a large proportion of people, between one and two-third on average, hear about or get their job through friends and relatives (see Pellizzari 2004, for cross-countries and cross-industries differences2). Moreover, to a large extent, employers also use social networks. Holzer (1987) reports that 36% of firms interviewed filled their last opening with referred applicants. Campbell and

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1The Hosios–Pissarides condition states that the decentralized equilibrium of a standard matching model with risk neutral agents is efficient if and only if the share of jobs surplus that accrues to the workers is equal to the elasticity of the matching function with respect to unemployment.

Mardsen (1990) find that about half of a sample of 52 Indiana establishments make regular use of referred applicants.

The aim of our paper is to analyze the job–worker matching process when workers and firms have access to different search methods. Given the importance of social networks in the actual job search process, it will be assumed that workers and firms can be matched together through social networks and through such alternative methods as private agencies and ads in newspapers, which can be more efficient, but also more expensive as a counterpart. Afterward, in 1995, Granovetter, who has written a similar book on the role of social networks in the job search process, argued that “a full understanding of matching requires also an assessment of when formal procedures have an advantage [over social networks], and when we may expect to find them. This is an important subject about which we know very little.” Our contribution is aiming at shedding some light on this issue. Our paper will indeed show that analyzing job search and job advertising in a context in which there are several search methods sheds some new light on the consequences and the efficiency of search activities in the labor market.

More precisely, the main contributions of our paper are the followings. First, we provide a very simple matching model of the labor market, inspired from Diamond (1981), Pissarides (2000), and Calvò-Armengol and Zenou (2005), with an endogenous arrival rate of job offers driven by free-entry. In this simple model, unemployed workers and jobs can be matched together through social networks and through more efficient, but also more costly, methods. Second, the choice of search methods is endogenous. This allows us to show that decentralized decisions to use social networks can be inefficient. Indeed, when an agent chooses which methods to use, he or she considers which methods are used by the other side of the market. For example, it is not useful for a firm to put ads in newspapers if very few workers use newspapers to find a job. In the same way, it is not helpful for a worker to buy newspapers to find a job if no firm uses this hiring channel. More generally, in some cases, the choice of search strategies suffers from coordination failures. It would be efficient for the economy that the agents choose another search strategy. However, a change on only one side of the market is not profitable. Hence, workers and firms must simultaneously modify their search strategies. In a decentralized equilibrium, the existence of imperfect information limits the possibility of coordination.

Consequently, with respect to an efficient allocation, in a decentralized equilibrium, social networks can be overutilized in some circumstances, and underutilized in others. As search costs with formal methods are higher, individuals overinvest in search activities if networks are underutilized. Hence, the existence of different job search methods can give rise to a greater job search intensity than the efficient one. This is in sharp contrast with the standard result, which has been recalled above, according to which search intensity is too low in matching models. Third, in our framework, a rise in unemployment benefits can induce a welfare improvement even if workers
are risk neutral. The study of the efficiency of search strategies offers new insights about the role of the public employment agency. Indeed it shapes the incentives to choose between different methods of search and can be used as a coordination device. For example, the existence of unemployment benefits, conditional on the use of formal methods of search, decreases the incentive to use social networks and could help to coordinate workers and firms on formal methods if they are more efficient.

The idea that social networks play a large and unusual role on the labor market has been explored for a long time by sociologists. In economics, some contributions have underscored that the transmission of job information through contact networks influences the job–worker matching process. To our knowledge, Boorman (1975) was the first to provide a formal network model to describe the information structure of job finding. In Boorman’s model, networks are endogenous: contacts are developed by individuals who maximize their probability of getting some new job in the event that they lose their present job. Boorman focused on the supply-side of the labor market only, whereas we rather study search strategies on both sides of the market. Accordingly, for the sake of simplicity, we assume that the network structure is exogenous. Calvò-Armengol and Zenou (2005) use Boorman’s framework to provide a matching model with contact networks and an endogenous arrival rate of job offers driven by free-entry. Our model allows us to explicitly derive equilibria with different search strategies, whereas Calvò-Armengol and Zenou (2005) do not analyze the issue of multidimensional search.

Holzer (1987) provides a model with an explicit choice of the search method together with instructive empirical work. Holzer considers a “partial” job search model where individuals face exogenously determined offer probabilities and wage offer functions that reflect the demand side of the market. Then, he assumes that search cost, offer probabilities and wage offer functions hinge on search methods. The model mainly aims at describing the tradeoff between the cost and benefit of each method for the workers. Moreover, the author provides empirical evidence of this tradeoff. However, in this paper, the hiring mechanism is largely “black boxed.” It does not investigate the equilibrium effects of the choice of search methods. Our model, focused on the matching process with an endogenous job arrival rate, allows us to consider explicitly the interactions between employers, and workers, search strategies.

In the same way, Montgomery (1991) examines the choice between alternative methods in an adverse selection model. In this framework, networks are used by firms to get information about the productivity of applicants. Since workers hired through networks are, on average, more productive, the wages

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3See Granovetter (1995) and the references therein.
4Fontaine (2008) also links the matching function with the social networks to study wage dispersion. Ioannides and Soetevent (2006) extend the model of Calvò-Armengol and Zenou (2005) to allow for randomness in the networks’ structure to generate wage and employment dispersion.
of the jobs got through networks are also higher than the wages on the “formal” labor market. Montgomery’s paper is focused on the effect of networks on the productivity of the matches. In our framework, all matches are equally productive. However, we study another feature of the existing tradeoff. Each method of search induces a different job arrival rate and has a different cost. Besides, we provide an equilibrium search model and thus study the efficiency of the decentralized equilibrium in an appropriate framework.

Wages are endogenous in our model. This last point is important, since it is well-known that networks have large effects on wages (see Ioannides and Loury 2004, Pellizzari 2004, Bentolila, Michelacci, and J. Suarez 2009). Calvó-Armengol and Jackson (2004, 2007) consider the effect of networks on wages and employment but in a different framework, more focused on the analysis of the effect of networks’ structure. Especially, they do not study either endogenous search strategies, or an equilibrium labor market. Bentolila et al. (2009) offer an equilibrium search model with social networks. They investigate, both empirically and theoretically, the effect of networks on wages in a labor market with heterogenous workers and firms. They find a negative wage premium for the use of networks. Additional evidence provided by Pellizzari (2004) and Loury (2006) shows that, more generally, the effect of the use of networks on wages is empirically ambiguous. With respect to this strand of the literature, our purpose is different. We aim at providing a framework where the choice between search methods is endogenous and study the effect of economic policies in that framework. We do not concentrate on the effect of networks on wages but on the efficiency of the labor market.

The model is presented in Section 2. Sections 3 and 4 are, respectively, devoted to the analysis of decentralized equilibria and efficient allocation. Section 5 provides some insights into the economic policy implications of the multidimensionality of search activities.

2. The Model

What happens when social networks compete with another method of search, more costly but more efficient (with respect to the job offer arrival rate)? Do workers choose the socially efficient method? In order to investigate these issues, we provide a simplified matching model of the labor market which borrows from Pissarides (2000), but where individuals make an explicit choice between networks and other methods. For the sake of simplicity, we first consider two types of symmetric decentralized equilibria: one where all workers and firms use social networks, one where they only use formal methods of search. In the next section, we study the stability of these equilibria by allowing workers and firms to change their search strategies.

2.1. The Labor Market

We begin by introducing notations. There is a large labor force, the size of which is denoted by \( L \), and an endogenously sized continuum of competitive
firms producing a numeraire output, using labor as sole input. Time is continuous, individuals are infinitely lived, risk neutral and discount the future at rate \( r > 0 \). There are \( U \) unemployed workers and we denote \( u \) the unemployment rate. An unemployed worker benefits from an income flow \( z \). Jobs are destroyed at an exogenous rate \( q \). The economy is at the steady state.

Our matching functions are based on simplified urn-ball processes (e.g. Albrecht, Gautier, and Vroman 2003, Albrecht et al. 2004), in which each worker has an urn and each job offer is a ball which is sent into urns. Vacancies send job offers at a rate \( a \). If there are \( V \) vacancies, our assumptions imply that the arrival rate of job offers amounts to \( Va \), and that the probability that more than one offer is sent by the finite set of \( V \) vacancies in a small interval of time \( dt \to 0 \) goes to zero (see the Appendix). In other words, the probability that an individual gets more than one job offer in a small interval of time, \( dt \to 0 \), goes to zero. Second, we assume that when an offer reaches a job seeker, the matching takes place immediately. Hence, an unemployed worker cannot have more than one offer at the same time when he moves out of unemployment. This assumption rules out the congestion effects pointed out by Albrecht et al. (2003, 2004, 2006)\(^5\) and simplifies the analysis. We denote by \( M = M(U, V) \) the matching rate. Of course, \( M \) depends on the agents’ search strategies.

In our continuous time framework, when all agents use the same method of search, the exit rate from unemployment is simply \( M/U \) and the hiring rate is equal to \( M/V \). Since our economy is at the steady state, the flow equilibrium of unemployment reads

\[
q (1 - u) L = M.
\]

We denote \( sc \) the unemployed workers’ search cost, which is a function of the search method, and \( w \) the wage. The value function of an unemployed worker, denoted by \( V_u \), satisfies:

\[
rv_u = z - sc + \frac{M}{U} (Ve - Vu)
\]

\( Ve \), the value function of an employee reads:

\[
rv_e = w + q (Vu - Ve).
\]

In the same way, the values of a vacant job and a filled job, denoted by \( \Pi_v \) and \( \Pi_e \), respectively, satisfy

\[
r\Pi_v = -hc + \frac{M}{V} (\Pi_e - \Pi_v)
\]

\[
r\Pi_e = y - w + q (\Pi_v - \Pi_e)
\]

\(^5\)In these papers, frictions occur because, in discrete time, some job seekers can receive more than one job offer (which could have been useful for other workers) and because firms can have more than one applicant at the same time (who could have filled other vacancies). See Stevens (2007) for a recent survey on the microfoundations of the matching function.
with \( hc \) the hiring cost, which hinges on the search strategy, and \( y \) the productivity of a job which is exogenously given.

Then, it is assumed that all profit opportunities from new jobs are exploited. Accordingly, jobs are created until the expected value of vacancies goes to zero: \( \Pi_v = 0 \). Finally, wages are bargained by firms and workers after they meet. The surplus of each match is shared according to the Nash solution to the bargaining problem. Let us denote by \( \beta \in [0, 1] \) the share that accrues to the worker. The Nash solution reads \((1 - \beta)(V_e - V_w) = \beta(\Pi_e - \Pi_v)\).

### 2.2. Decentralized Equilibrium with Costly Search Methods

Formal search methods are supposed to be the most effective but also the most costly search methods. Remember that we do not try to model all “formal” search methods but rather to consider a case where there exists a method of search which entails a higher job arrival rate but which is more costly. Formally, in our model, a worker can “switch his urn on,” at cost \( sc = \bar{s} \) per unit of time, to signal that he is looking for a job. Similarly, an employer pays \( hc = \bar{c} \), in order to check, in the entire population, if some urns are switched on. Hence, when both sides of the labor market use this method of search, the matching rate \( M \) is simply \( Va \), the job arrival rate. To put it simply, formal methods of search do not suffer from congestion effects or informational problems. This represents in a very stylized way the formal methods of search: firms and workers are engaged in costly search activities, which make job–worker matches easier.

Using the values functions, the free entry condition and the Nash bargaining, one gets

\[
\frac{\bar{c}}{a} = \frac{(1 - \beta)(y - z + \bar{s})}{r + q + \beta q \frac{(1 - \bar{u})}{\bar{u}}}. \tag{1}
\]

This equation shows that, in equilibrium, the expected cost of a vacancy, represented by the left-hand side, equals the expected profit of a filled job, represented by the right-hand side. It defines a unique value of \( \bar{u} \in [0, 1] \) if \( \bar{c}/a < (1 - \beta)(y - z + \bar{s})/(r + q) \). One gets \( \bar{u} = 1 \) if \( \bar{c}/a \geq (1 - \beta)(y - z + \bar{s})/(r + q) \). The equilibrium is displayed in Figure 1, in which the expected cost of a vacant job, \( \bar{c}/a \), is represented by the \( EV \) schedule and the expected profit of a filled job is represented by the \( EP \) schedule.

### 2.3. Decentralized Equilibrium with Social Networks

The alternative method relies on social networks. In this case, workers’ search cost \( sc \) is assumed to amount to zero. With formal methods, individuals have to put ads in newspapers, send direct applications, register with private or public agencies, whereas networks lead generally to a lower cost (Granovetter 1995)
since workers do not actively search.\footnote{For instance, Hansen and Pratt (1991) claim that the “fixation of the traditional job search model on the cost of acquiring information [. . .] seems misplaced, as for the majority of cases in our sample, the information leading to new job came through existing social networks at no cost to the worker at all.” Even if this assertion does not take into account the cost of maintaining a network of friends and relatives, such studies give some empirical basis to our assumption.} Similarly, employers who do not pay to check which urns are switched on, can send their offers at random into the urns at a low cost $hc = c < \bar{c}$. Accordingly, job offers can reach an unemployed worker but also an employee. A large part of the empirical literature on networks bears the assumption that $c < \bar{c}$. For instance, Fernandez, Castilla, and Moore (2000) find that using networks (in the form of a referral bonus) leads to an important reduction in recruiting costs.

When an offer is received by an employee, we assume that he forwards this offer through his network of acquaintances. If there is more than one unemployed worker in the network, he chooses one at random. Moreover, note that there exists a positive probability that the employee has no unemployed friends. In this case, the offer is lost.

We now define the social networks. First, they are supposed to be exogenous. Although it would be interesting to understand why employees are willing to help other workers to find a job, this is beyond the scope of this article. On the one hand, the expected payoff of cooperation could relate to future job search (for example in case of a job loss). On the other hand, it can also be related to something “outside” the labor market. For example, it could be affected by social conventions, a social group’s traditions or

\[ \frac{(1-\beta)(y-z+\bar{c})}{(r+q)} \]

Figure 1: The equilibrium without social network.
social prestige. Besides, for the sake of simplicity, networks are supposed to be identical across workers. All workers have the same numbers of social links and a network links a worker with $\ell$ other workers. It is also assumed that information is transmitted only to the direct links. In other words, if $i$ and $i'$ are linked, and $i''$ is linked to $i'$, but not to $i$, the information gotten by $i'$ can be transmitted to $i$ and $i''$. However, the information gotten by $i$ can be transmitted to $i'$, but not to $i''$. The networks we consider are thus very simple. Other papers, e.g., Ioannides and Soetevent (2006), consider richer structures. However, we are not interested in the effect of the heterogeneity of the networks’ structure on wage/employment dispersion, but rather on the efficiency of the decentralized economy when workers and firms choose between alternative search methods.

Given this set of assumptions, we can derive the matching rate in an economy where workers and firms use social networks. When a job offer reaches an unemployed worker, an event that occurs with probability $u$, the unemployment rate, the job is immediately filled. If it reaches an employee, an event that occurs with probability $(1 - u)$, the employee forwards it to the unemployed workers who belong to his/her network. If there is more than one unemployed worker in the network, they all have the same probability to obtain the offer.

As any individual of any network is employed with probability$^{7}(1 - u)$, the probability that all the $\ell$ links of any individual are employed amounts to $(1 - u)^\ell$. Hence, the probability that a job offer sent at random into $L$ urns reaches an unemployed worker reads:

$$\frac{u}{\text{Probability to reach an unemployed worker}} + \frac{(1 - u)}{\text{Probability to reach an employee}} \cdot \left[1 - (1 - u)^\ell\right] = [1 - (1 - u)^{\ell+1}]$$

Eventually, given that job offers arrive at rate $Va$, job offers meet job seekers at rate

$$M = Va[1 - (1 - u)^{\ell+1}].$$

$^{7}$It should be noticed that this result holds only if each individual is rematched at random with $\ell$ links just after every job arrival. Otherwise, there would be a distribution of unemployment rates across networks. Taking into account such a phenomenon is beyond the scope of this paper. We adopt the standard (and often implicit) assumption that each individual is re-matched at random with $\ell$ links just after every job arrival (Diamond 1981, Calvò-Armengol and Zenou 2005).
Hence, using the free-entry condition and the Nash bargaining, the equilibrium unemployment rate, denoted by $u_\ell$, satisfies

$$\frac{c}{a[1 - (1 - u_\ell)^\ell + 1]} = \frac{(1 - \beta)(y - z)}{r + q + \beta q (1 - u_\ell) u_\ell}.$$  (3)

It can easily be verified that this equation defines a unique equilibrium value of the unemployment rate in the interval $[0, 1]$. The equilibrium is displayed in Figure 2.

Note that the unemployment rate decreases with the network’s size. Large networks are favorable to employment because they entail low expected hiring costs, as is shown by the left-hand side of Equation (3) which decreases with $\ell$.

3. The Stability and the Efficiency of Decentralized Equilibria

In the previous section, we derived two types of symmetric decentralized equilibria. In this section, we investigate their stability. We show that multiple equilibria can arise: both equilibria can be stable for the same parameter values. For the sake of simplicity, we use a very simple concept of stability. An equilibrium is stable if it is not profitable for a single agent to deviate. This simple definition helps to deliver tractable results and to shed light on inefficiency in the choice of search methods. This inefficiency comes from coordination failures between workers’ and firms’ search strategies.
3.1. The Decision to Use Social Networks

Note that, in the previous section, we only consider symmetric equilibria. To study the stability of these equilibria, we need to define what happens if an employer decides to use formal methods of search while no workers use these methods. We could assume that, in this case, the offer is lost. We do not make this assumption. We assume that, since the employer does not see any urn switched "on," he or she sends the offer at random. If the offer is received directly by a job seeker, s/he is hired immediately. Otherwise, if the offer is received by an employee, it is forwarded to an unemployed friend of this employee. If there is not any unemployed worker in the network, the offer is lost.

We now consider a symmetric equilibrium with social networks. It is stable only if each worker and employer prefers to use social networks rather than trying to get a match thanks to the alternative method. In case of deviation from an equilibrium with networks, an employer can decide to use a costly search method, instead of relying only on the information transmitted by the employees. However, in our simple model, it is never worth paying high search costs to check whether there are some urns switched on by unemployed workers as all urns are off when no worker pays high search cost. Therefore, employers never deviate from an equilibrium with low search costs. The same reasoning implies that unemployed workers have no incentive to deviate from a stable equilibrium in which no employer pays high search costs, because it is not worth bearing the cost of switching one’s urn on if nobody pays the cost of checking whether some urns are on. Hence, the use of social networks is always a stable equilibrium.

3.2. The Decision to Use Costly Search and Hiring Methods

Consider an equilibrium where workers and firms use formal methods of search. An employer always has the possibility to deviate and rely on networks of employees to find a worker. In order to look at the deviation from an equilibrium in which costly search methods are utilized, we begin by defining the value of a vacant job in an equilibrium with high search costs, denoted by \( \tilde{\Pi}_v \). To make the timing of the decisions clear, we consider this value at the period \( t \):

\[
\tilde{\Pi}_v(t) = \frac{1}{1 + r dt} \left[ -\tilde{c} dt + \frac{M}{V} dt \tilde{\Pi}_e(t + dt) + \left[ 1 - \frac{M}{V} dt \right] \max \left\{ \tilde{\Pi}_v(t + dt), \tilde{\Pi}_d(t + dt) \right\} \right],
\]

where \( \tilde{\Pi}_e \) denotes the value of a filled job in stable equilibrium without social networks and \( \tilde{\Pi}_d(t) \) the value of a vacant job when the employer deviates and uses social networks. In case of deviation from equilibrium with high search costs,
costs, the probability to reach an unemployed worker when a job offer is sent amounts to \([1 - (1 - \bar{u})^{\ell+1}]\). This implies that \(\Pi_v^d(t)\) satisfies
\[
\Pi_v^d(t) = \frac{1}{1 + r\Delta t} \left[ -cdt + a[1 - (1 - \bar{u})^{\ell+1}] dt\Pi_e(t + dt) \right. \\
\left. + \{1 - a[1 - (1 - \bar{u})^{\ell+1}] dt\} \text{Max}\{\Pi_v(t + dt), \Pi_v^d(t + dt)\} \right].
\]

Using the free-entry condition, one gets \(\Pi_v(t + dt) = 0\) and \(\Pi_e(t + dt) = c/a\), which implies that a deviation from equilibrium with high search costs is not profitable if \(\Pi_v(t) \geq \Pi_v^d(t)\), which is equivalent to:
\[
\frac{c}{[1 - (1 - \bar{u})^{\ell+1}]} - \tilde{c} \geq 0. \tag{4}
\]

Condition (4) shows that the use of a costly search and hiring method can be a stable equilibrium only if the relative cost of this method is low, and if the size of the networks is sufficiently small. For large values of \(\tilde{u}\) the deviation becomes more profitable since the probability that an employee has no unemployed friend is low. Accordingly, this condition hinges on the parameters which influence the equilibrium unemployment rate \(\bar{u}\): firms are more likely to deviate from this equilibrium if the workers’ search cost, \(\bar{s}\), is small, if the income of unemployed workers, \(z\), and the bargaining power of employees, \(\beta\), are large.

Let us now look at the strategies of unemployed workers. If \(\tilde{V}_u(t)\) stands for the expected utility of an unemployed worker in an equilibrium where costly search methods are used and \(\tilde{V}_u^d(t)\) stands for the expected utility of an unemployed worker who deviates from such an equilibrium, one gets
\[
\tilde{V}_u(t) = \frac{1}{1 + r\Delta t} \left[ (z - \bar{s}) dt + \frac{M}{\bar{u}L} dt\tilde{V}_e(t + dt) \right. \\
\left. + \left[1 - \frac{M}{\bar{u}L} dt\right] \text{Max}\{\tilde{V}_u(t + dt), \tilde{V}_u^d(t + dt)\} \right],
\]
\[
\tilde{V}_u^d(t) = \frac{1}{1 + r\Delta t} \left[ zdt + \text{Max}\{\tilde{V}_u(t + dt), \tilde{V}_u^d(t + dt)\} \right],
\]
where \(V_e(t)\) is the expected utility of an employee at date \(t\) in the equilibrium with costly search methods. Notice that deviating from the “formal” equilibrium implies that the deviator remains unemployed. Then, a deviation from stable equilibrium with formal search methods is not profitable to the unemployed workers if \(\tilde{V}_u(t) \geq \tilde{V}_u^d(t)\). In steady state equilibrium, this condition is equivalent to
\[
\tilde{s} \leq \frac{M}{\bar{u}L} (\tilde{V}_e - \tilde{V}_u). \tag{5}
\]

Note that, at the steady state, \(M = Va = q(1 - \bar{u})L\), and that the sharing rule together with the free entry condition imply that \((1 - \beta) (\tilde{V}_e - \tilde{V}_u) = \beta \tilde{c} / a\). Using the definition of \(\bar{u}\), Equation (5) can be written as
This last condition defines the set of parameter values that ensure that workers do not deviate from an equilibrium in which costly search methods are used. It deserves at least two comments.

First, it appears that an increase in workers’ bargaining power incites workers to deviate from the equilibrium with costly search methods. At first glance, this might seem surprising, since one might think that unemployed workers search harder if they get larger rents when they find a job. Actually, in equilibrium, the returns to formal methods decreases with the bargaining power of workers. Indeed, an increase in the workers’ bargaining power implies a rise in \( \tilde{u} \) and, consequently, a decrease in the exit rate from unemployment. Therefore, unemployed workers deviate from the equilibrium with costly search methods when the bargaining power of employees is important.

Second, condition (6) does not depend on the cost of formal search methods for the unemployed workers, \( \tilde{c} \). This comes from the fact that \( \tilde{V}_u \) does not depend on \( \tilde{s} \) at equilibrium. When \( \tilde{s} \) increases, unemployed workers bear larger search costs, then wages decrease through the bargaining process, which induces firms to create more vacant jobs that exactly compensate the impact of the variation in \( \tilde{s} \) on \( \tilde{V}_u \).

3.3. The Multiplicity of Equilibria

Firstly, recall that firms and workers have no incentive to deviate from an equilibrium with social networks. Hence, an equilibrium with low search costs is sustainable for any set of parameter values satisfying \( \tilde{s} > s = 0 \) and \( \tilde{c} > c \). The use of costly search methods can be sustained as a stable equilibrium if and only if conditions (4) and (6) are fulfilled. Thus, a multiplicity of equilibria can occur, as the social networks and the alternative methods can be equilibria for the same set of parameter values. The multiplicity of equilibria is displayed in the \((\tilde{s}, \beta)\) plane in Figure 3. In the area \( N \), the situation in which social networks are used is a stable equilibrium. In the area \( F \), where (4) and (6) are fulfilled, the situation in which costly search methods are used is a stable equilibrium. In the area where multiple stable equilibria exist, the unemployment rate is always larger if individuals use social networks rather than costly search methods. By definition of the matching processes, formal methods induce less friction on the labor market.

Figure 3 shows that the utilization of costly search methods can be a stable equilibrium if the bargaining power of workers, \( \beta \), is small enough and if the search cost of workers, \( \tilde{s} \), is large enough. That costly search methods can be sustained as a stable equilibrium for large values of the search cost of workers only might seem counter-intuitive. Actually, the explanation relies on the fact that condition (6), which ensures that workers do not deviate from the equilibrium with costly search methods, does not hinge on \( \tilde{s} \). Accordingly,
large values of $\bar{s}$, that contribute to lower the unemployment rate $\bar{u}$, also decrease the returns to networks as a recruitment channel, and eventually incite firms to prefer costly search methods rather than networks as a way of hiring workers.

The multiplicity of equilibria relies on some form of strategic complementarity. Formal methods of search will be used by employers (workers respectively) only if some workers (some employers respectively) use the same method of search. Clearly, our concept of stability is very simple since we only consider the possibility of the one-shot deviation of a single agent. In a more general framework where firms could anticipate the deviation of some of the job seekers if they deviate, the use of social networks would no longer always be an equilibrium. However, if there is some uncertainty about the fact that job seekers would react in the next period by deviating too, or if there are difficulties for the firms to locate the workers who deviate, multiple equilibria could still arise. Since information is imperfect, this uncertainty is likely to exist in the real labor market. Workers rarely observe directly the method used by the firms. A firm which deviates needs a worker to take note of its deviation. For example, the firm needs some workers to check the local newspapers or go to the employment agency. It is unlikely to be immediate.
and there is a positive probability that the deviation is unsuccessful. To put it simply, in a more general framework, a firm which deviates and decides to use the informal search method will not benefit from a hiring rate of \( a \) but \( a \times \varepsilon \) with \( 0 < \varepsilon < 1 \). Hence, the deviation from an equilibrium with social networks will not necessarily be profitable, even if an equilibrium with the formal methods exists for the same values of the parameters.

Generally speaking, our simple framework builds on simplifying assumptions in order to encompass two stylized facts. First, the cost of searching through social networks is very low and another method will be used only if the increase in the matching rate balances the additional search cost. Second, the profitability of a deviation is clearly related to the extent of its use by the other side of the market. The complementarity induced by our very simple definition of stability is present in the real labor market. The higher the number of employers using a given method of search, the more profitable it is for a job seeker to use this search method. In the same way, the higher the number of job seekers using a search method, the more profitable it is for an employer to use this method to fill a vacancy. Considering this argument, the main difference between the formal methods and the networks is that the formal methods need an additional investment on both sides of the market.

The comparison of the efficiency of multiple equilibria relies on the analysis of the sign of the externalities induced by the search decisions of workers and firms. This analysis is far from being trivial in a framework in which the decisions of each individual exert an influence on both sides of the market. Accordingly, it is worth looking in some detail at the efficiency issue to shed light on this problem.

4. Social Efficiency

This section is devoted to the analysis of efficient allocation and its comparison with decentralized equilibrium. For the sake of simplicity, we limit the analysis to symmetric allocations in which all unemployed workers and all employers choose the same search strategy. We begin by defining the efficient allocation, first in the economy with social networks, and second in the economy in which social networks are not used as a channel to match workers and jobs. Finally, we determine the values of the parameters such that it is efficient to use networks in the job search process.

4.1. The Efficient Outcome with Social Networks

The social planner chooses the vacancy rate at each date \( t \), denoted by \( v(t) \), which maximizes the discounted value of the stream of production of the numeraire output per individual, which amounts to: \( y[1 - u(t)] + u(t)z - v(t)c \). The value function of the social planner, denoted by \( W[u(t)] \), satisfies...
\[ W[u(t)] = \max_{v(t) \geq 0} \left( \frac{1}{1 + rdt} \right) \{[y[1 - u(t)] + u(t)z - v(t)c]dt + W[u(t + dt)] \} \]

subject to: \[ u(t + dt) = u(t) + [1 - u(t)]q dt - M dt. \]

Let us denote by \( M_i, i = v, u, \) the partial derivatives of the matching function with respect to \( v \) and \( u \), respectively. The first-order and the envelope conditions read, respectively

\[-c - M_v W'[u(t + dt)] = 0 \]
\[(z - y) dt + W'[u(t)] = W'[u(t + dt)][1 - q dt - M u dt]. \]

In a steady-state, these two conditions imply, together with the law of motion of \( u(t) \) and the definition of \( M \) in the social networks case, that the equilibrium value of the efficient unemployment rate with social networks, denoted by \( u^*_\ell \), satisfies

\[
\frac{c}{a[1 - (1 - u^*_{\ell})^{\ell+1}]} = \frac{(y - z)}{r + q + q \frac{(\ell + 1)(1 - u^*_{\ell})^{\ell+1}}{1 - (1 - u^*_{\ell})^\ell}}. \tag{7}
\]

The comparison of the decentralized equilibrium value of the unemployment rate with social networks, \( u^*_\ell \), defined Equation (3), with the efficient solution, \( u^*_{\ell} \), defined Equation (7), shows that the decentralized equilibrium is efficient if and only if

\[
\beta = \beta^* = \frac{M_u}{r + q + M u^*_\ell \left(1 + \eta(u^*_\ell)\right)} \in ]0, 1[, \text{ where } \eta(u^*_\ell) = u^*_\ell \frac{M_u}{M}. \tag{8}
\]

This condition looks like the Hosios–Pissarides condition (Hosios 1990, Pissarides 2000) which states that the decentralized equilibrium of a search and matching model is efficient if and only if the share of the surplus of the jobs that accrues to the workers, \( \beta \), is equal to the elasticity of the matching function with respect to the unemployment rate in a context in which the matching function is homogeneous of degree one with respect to \( u \) and \( v \). In our framework with social networks, the matching function is not homogeneous with respect to \( u \) and \( v \). But there exists a value of \( \beta \) which makes the decentralized equilibrium efficient.

### 4.2. The Efficient Outcome with Costly Search Methods

When costly search methods are used, the efficient outcome can be found very easily. The expected cost of a vacant job amounts to \( \bar{c}/a \), and the expected gain from a filled job is \( (y - z + s)/(r + q) \). Both values hinge neither on
unemployment rate nor on the number of vacancies. Accordingly, it is worth creating vacancies only if $\bar{c}/a \leq (y-z+\bar{s})/(r+q)$. If this condition is not fulfilled the efficient unemployment rate is equal to one. If this condition is satisfied, it is optimal to create vacancies until the unemployment rate goes to zero, which implies that the efficient vacancy rates, $V/L$, amounts to $q/a$. In other words, the efficient unemployment rate without social networks, denoted by $\bar{u}^*$, satisfies

\begin{equation}
\bar{u}^* = \begin{cases} 
0 & \text{if } \bar{c}/a \leq (y-z+\bar{s})/(r+q) \\
1 & \text{otherwise.}
\end{cases}
\end{equation}

(9)

The comparison of the efficient unemployment rate, $\bar{u}^*$, with the unemployment rate obtained in the decentralized equilibrium, $\bar{u}$ (see Equation (1)), shows that the decentralized equilibrium is efficient if and only if $\beta = 0$ in the nondegenerate case, $\bar{c}/a \leq (y-z+\bar{s})/(r+q)$, which is compatible with positive employment.

4.3. The Efficient Use of Social Networks

Let us denote by $\bar{W}(u)$ the value function of the social planner when costly search methods are utilized. Assuming that the condition to get $\bar{u}^* = 0$ is fulfilled (see Equation (9)), the value function of the social planner amounts to $\bar{W}(0) = [y-(\bar{c}q/a)]/r$ at the optimum. The optimal value of the objective of the social planner when social networks are used amounts to $W(u^*_\ell) = [y(1-u^*_\ell)+u^*_\ell z - cu^*_\ell]/r$. It is efficient to use costly search methods if $W(u^*_\ell) \leq \bar{W}(0)$, which is equivalent to

\begin{equation}
\frac{c}{1-(1-u^*_\ell)^{\ell+1}} - \frac{c}{a[1-(1-u^*_\ell)^{\ell+1}]} \geq 0.
\end{equation}

(10)

Equations (4), (6), and (10) show that decentralized decisions do not generally yield an efficient use of social networks.

This issue can be clarified by comparing the decentralized equilibrium with the efficient allocation for different values of the bargaining power of employees. Notice that, in an economy with formal methods of search, as long as $u > 0$, the job filling rate does not depend on the unemployment rate. On the contrary, the higher the unemployment rate, the lower the probability that an offer is received by a network without any unemployed workers in an equilibrium with social networks.

Hence, if $\beta$ is sufficiently large, networks are the only stable equilibrium. In these cases, the wages and thus the unemployment rate are sufficiently large to make the equilibrium with formal methods unstable. As condition (10) does not hinge on $\beta$, this implies that costly search methods can be underutilized in the decentralized equilibrium for some parameter values when workers have significant bargaining power. In the opposite situation,
in which the bargaining power of workers is low, social networks can be underutilized if individuals are coordinated on a stable equilibrium with costly search methods.

It is worth noting that if the coordination failure leads to an underutilization of social networks, both unemployed workers and firms devote too much resources to the search activity. Accordingly, we are in a situation in which job search intensity is too high. This is in sharp contrast with the standard result usually obtained in search and matching models, according to which labor market frictions and the inefficiency of the decentralized equilibrium induce workers to adopt too low a search intensity (Mortensen 1982, Pissarides 2000). When the workers’ bargaining power is too low, the unemployment rate is too low in comparison with the efficient one, as in a standard matching model. However, this has a very special effect in our framework. When the unemployment rate is very low, the formal methods of search become very attractive. The employers do indeed pay a high search cost but can find an applicant even if there are only few unemployed. On the contrary, if an employer uses the social networks, he has a high probability that his offer is received by a network without any unemployed. If the bargaining power is low enough, the equilibrium with the formal methods is stable. Remember that the efficient choice of the search method by a social planner does not hinge on the bargaining power but on the structural parameters of the economy. Hence, for very low value of $\beta$, the formal search equilibrium is stable although the allocation can be inefficient.

In our framework, which takes account of the existence of different search strategies, workers and firms can be badly coordinated on expensive search strategies that lead them to choose an inefficiently high search intensity. Obviously, it can also be the case that networks are overutilized. Such a coordination failure leads to a high level of unemployment. This case might illustrate some features of European Mediterranean countries in which networks provide an important source of insurance against unemployment thanks to transfers between agents that do not involve only standard economic assets, but also jobs (see Bentolila and Ichino 2008). It could be the case that the high unemployment rates in these countries result from an overutilization of networks at the expense of more efficient matching methods.

5. Economic Policy

In the preceding sections we saw that equilibrium unemployment can be inefficient because individuals choose wrong search methods. In this section, we ask whether some policy tools could reduce this inefficiency. In this perspective, two issues are discussed. First, it appears that unemployment benefits, through their influence on the search strategies of firms and workers, have nonstandard effects on welfare. Namely, an increase in unemployment benefits can improve efficiency by inciting individuals to coordinate on efficient search methods. Second, and more important, we consider unemployment
benefits and job offers subsidies *conditional on the search method*. Such conditional search subsidies allow the policy maker to force firms and workers to deviate from an inefficient equilibrium. More precisely, we have seen that one cannot deviate from an equilibrium with job search relying on social networks, even if this method is inefficient. From this point of view, it can be useful to use coordination devices in order to create incentives to adopt costly methods. Our framework allows us to show that conditional unemployment benefits and search subsidies for firms can play an important role in this realm.

### 5.1. The Consequences of Unemployment Benefits

Let us assume that unemployed workers get unemployment benefits, denoted by $b$, that are financed by lump-sum taxes. Simple replication of the reasoning above implies that Equations (3) and (1), which define the decentralized equilibrium unemployment rate, can be rewritten as follows:

\[
\begin{align*}
\frac{c}{a [1 - (1 - u_e) \ell + 1]} &= \frac{(1 - \beta) (y - z - b)}{r + q + \beta q (1 - u_e)} & \text{if networks are used} \\
\frac{\bar{c}}{a} &= \frac{(1 - \beta) (y - z - b + \bar{s})}{r + q + \beta q (1 - \bar{u})} & \text{if formal methods are used.}
\end{align*}
\]

Equation (11) shows that rises in $b$ always lead to increases in the unemployment rate, through a wage pressure rise. Nevertheless, unemployment benefits hikes can have a positive impact on efficiency if they force individuals to give up coordination on an inefficient equilibrium in which costly search methods are used. As already mentioned, in a equilibrium with formal methods, the hiring rate does not depend on the unemployment rate and is simply $a$.\(^8\) On the contrary, the matching rate with social networks is an increasing function of the unemployment rate. The increase in the unemployment benefits induces a rise in the unemployment rates in both types of equilibrium. However, the returns to the use of informal methods for the firms increases with the unemployment rate. Beyond some value of $b$, the relative effectiveness of the formal method does not balance the additional search cost anymore. The equilibrium with social networks becomes the only stable equilibrium.

Formally, let us begin by remarking that conditions (4) and (6) which define the range of parameters values for which the costly search methods can

\(^8\)Of course, this is due to our assumptions about the matching technology. However, in a more general model, the comparative advantage of the “costly” methods would still be, for a given level of unemployment, a higher hiring rate. Hence, their relative effectiveness as a hiring channel would still decrease with the unemployment rate.
be sustained as a stable equilibrium still hold when unemployment benefits are introduced. It appears that condition (6) that ensures that workers do not deviate does not hinge on \( b \). However, according to condition (4), firms are more likely to deviate from an equilibrium with costly search methods if \( b \) is increased, since \( \bar{u} \) increases with \( b \). Then, looking at Figure 4, it can be seen that an economy at point \( E \), where costly search methods can be sustained as a stable equilibrium, that faces an unemployment benefit rise, can switch to a situation in which costly search methods cannot be used any more in a decentralized equilibrium. As costly search methods can be overutilized in the decentralized equilibrium—for instance, if the bargaining power of employees is small, it can be the case that unemployment benefit hikes force individuals to switch from a “bad” equilibrium, in which too much resources are devoted to search, to a “good” equilibrium, in which job–worker matches rely on networks.

This case is illustrated by Figure 5, which displays the relation between unemployment benefits, the unemployment rate, and welfare. There is a discontinuity in the unemployment rate and welfare schedules around the point \( b = 0.1 \), where the economy goes from an equilibrium in which formal methods are used to an equilibrium in which employers recruit through
social networks. The increase in the unemployment rate is large. However, welfare rises simultaneously. Obviously, this example is only illustrative, especially since we only consider symmetric equilibria. Nevertheless, it stresses that changes in search strategies may strongly influence the relation between unemployment benefits and welfare.

5.2. Conditional Unemployment Benefits and Job Offers Subsidies

Let us now assume that, on top of their income flow $z$, unemployed workers receive unemployment benefits, denoted $b$, on the condition that they use costly search methods. For instance, in order to receive unemployment benefits, an unemployed worker must go regularly to a public employment agency and accepts interviews. Similarly, when firms are looking to fill a vacant job, they receive a subsidy, $h$, per unit of time, on the condition that they use formal search methods. In the real world, this subsidy could be conditional on the fact that firms send their offers to public employment agencies.

These transfers are assumed to be financed through lump sum taxes. Accordingly, taxes modify neither surpluses nor the surplus sharing rule. Thus, simple replication of the reasoning above implies that Equations (3) and (1), that define the decentralized equilibrium unemployment rate, can be rewritten as follows:

\[
\begin{align*}
\frac{c}{a \left[1 - (1 - u) \ell + 1\right]} &= \frac{(1 - \beta)(y - z)}{r + q + \beta q \frac{1 - u}{u}} \quad \text{if networks are used}, \\
\frac{\tilde{c} - \tilde{h}}{a} &= \frac{(1 - \beta)(y - z - b + \bar{z})}{r + q + \beta q \frac{1 - \bar{u}}{\bar{u}}} \quad \text{if formal methods are used.}
\end{align*}
\]

Can public employment agencies make stable equilibria no longer sustainable when they are inefficient?

It can be shown that it is possible to incite individuals to abandon networks thanks to conditional search subsidies on both sides of the market.
First, it is straightforward to see that if the agency gives conditional unemployment benefits $b$ to unemployed workers, with $b > \tilde{s}$, workers always choose costly search methods. Indeed, unemployment benefits are sufficiently high to cover the additional search expenses. Moreover, our assumptions about the search technologies imply that, if no employers use formal methods of search, workers can still receive offers through their network. In a similar way, firms always prefer to use costly search methods if the agency gives a conditional subsidy $h$, with $h > \tilde{c} - c$. Therefore, the use of costly search methods is the only stable equilibrium if $b > \tilde{s}$ and $h > \tilde{c} - c$.

This line of reasoning shows that a public agency which “buys” the job demands and offers at prices $b > \tilde{s}$ and $h > \tilde{c} - c$, respectively can coordinate individuals on an equilibrium with a low unemployment rate, as costly search methods entail a lower unemployment rate than the utilization of social networks. From this point of view, conditional unemployment benefits together with job offers subsidies allow individuals to adopt more efficient search methods, and lead to decreasing the unemployment rate. Of course, it is worth using conditional search subsidies only if the costly search method is more efficient than the utilization of social networks.

This case arises if condition (10) is fulfilled. In this situation, we have seen that, if $\tilde{c}/a \leq (y - z + \tilde{s})/(r + q)$, the social planner posts vacancies until $\tilde{u} = 0$. It is easy to define the economic policy that allows us to implement social efficiency. Equation (12) defines the unemployment rate when formal methods of search are used. One can verify that, when $b = \tilde{s}$ and $h = \tilde{c}$, one gets $\tilde{u} = 0$. As $h = \tilde{c}$, the instantaneous cost of vacancies amounts to zero, which implies that the number of vacancies created by firms can go to infinity. Such a situation is obviously inefficient. Accordingly, the public agency should limit the number of job offers that benefit from subsidies. The flow equilibrium implies that this number has to be equal to $q/a$ in order to entail zero unemployment.

It is worth noting that some empirical evidence supports our analysis. Van den Berg and van der Klaauw (2006) investigate the consequences of conditional subsidies using a controlled social experiment. One of their results is that unemployment benefits conditional on the use of formal methods induce a shift from informal to formal job search. However, the exit rate from unemployment remains unaffected. On the basis of Gorter and Kalb’s work (1996), the authors argue that their data take into account only relatively high skilled workers in favorable macroeconomic conditions. Such workers have very efficient social networks and consequently the deviation from an equilibrium where they overutilize the informal methods has only a small impact on the exit rate from unemployment.9 Accordingly van den Berg and

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9Our model is not designed to address the issue of the ex ante efficiency of the network. However, let us remark that when the probability that an employee has not any unemployed worker in his network goes to zero (that is when $\ell \rightarrow L$), the two methods yield the same job offers arrival rate $Va$. 
van der Klaauw present some evidence, found in the empirical literature, that for low skilled individuals with weak social networks, conditional subsidies could affect positively the exit rate from unemployment.

In conclusion, it appears that our results shed some new light on the role of employment agencies. First, in a context where individuals have access to different search methods, employment agencies are an instrument of coordination and, therefore, can contribute to eliminate inefficient outcomes of a decentralized labor market. This is an important difference from the usual approach to the design of unemployment insurance, according to which the unemployment agency is considered as a principal that tries to provide the appropriate incentives in order to raise the agent’s search intensity (see, for instance, Boone and van Ours 2006). In our model, subsidies are considered as a device to coordinate on the efficient method of search. It should be remarked that the choice of search methods can be easily monitored. Moreover, formal search methods mostly give rise to observable actions. Indeed, since the firms must send their offers to a public employment agency if they want to get subsidies, the agency observes immediately their search effort. In the same way, workers are urged to make an observable effort (for example to post their CV and accept the interviews provided by the agency) to receive unemployment benefits. Besides, we have shown that it is necessary to give conditional subsidies to both sides of the market. Conditional unemployment benefits can reduce the unemployment rate and increase welfare, if firms also get search subsidies. From this perspective, our contribution shows that it can be fruitful to take into account the interactions between different policy tools to achieve labor market efficiency.

6. Discussion

Recent literature shows a growing interest in social interactions and argues that they can explain a wide range of phenomena that was previously omitted in economics. We have attempted to provide a simple framework to study some effects of social networks on labor market equilibrium. We have developed a simple matching model in which unemployed workers and employers can be matched together either through social networks or through more efficient, but also more costly, methods. In this framework, the use of social networks is an endogenous choice and we show that the decentralized choices of search strategies can give rise to multiple equilibria for some values of the parameters.

It is also shown that decentralized decisions can be inefficient. The existence of different job search methods can give rise to a job search intensity which is higher than the efficient one. This is in sharp contrast with the standard result according to which search intensity is always too low if not efficient. Moreover, these results give a theoretical answer to Granovetter (1995) when he enquires into the efficiency of different job search methods. Then, we ask whether some policy rules could reduce the inefficiency
of the decentralized equilibrium with multidimensional search strategies. We consider two instruments: unemployment compensation and search subsidies for firms conditional to the method used. Both methods can lead to a more efficient coordination of firms. More surprisingly, for some values of the parameters, a rise in unemployment compensation induces a decrease in the unemployment rate. If the decentralized equilibrium is characterized by an inefficient use of social networks, this rise can force workers to coordinate on a more efficient method, which induces a lower unemployment rate. From this point of view, our analysis sheds new light on the role of public employment agencies. They are an instrument of coordination between workers and employers and can steer the economy away from inefficient equilibrium.

Our results have been obtained with a very simple model. Some extensions could lead to a better understanding of the consequences of social networks on the labor market. First, the network size is exogenous. It would be more appropriate to suppose that workers can adjust their network in order to respond to changes in labor market tightness. An endogenous choice of search strategies by workers could induce some interesting labor supply effects that our framework omits. Moreover, we have assumed that networks are continuously recomposed and that workers do not know whether the members of their network are employed. This implies that the expected gains of workers do not hinge on the actual composition of their network. By assuming more rigid network structures, we would get a situation in which workers embedded in networks with few employees have a smaller probability to move into employment than workers belonging to networks with many employees (Calvò-Armengol and Jackson 2004, Ioannides and Soetevent 2006, Fontaine 2008). To some extent, these mechanisms may contribute to influence the wage distribution and inequalities. Finally, an empirical estimation of the coordination failures pointed out by our paper would need a richer framework where individuals can use different methods in the same time, with different intensities. These issues are on our research agenda.

Appendix

A.1. The Arrival Rate of Job Offers

This appendix presents standard results about the properties of the arrival rate of job offers (see, e.g., Ross 1985). They are given for the sake of clarity.

Let us assume that there are $V$ vacancies. Every vacancy sends job offers according to a Poisson process with arrival rate $a > 0$. Let us denote by $N(dt)$ the number of job offers sent by a given vacancy in the time interval $dt$. According to the definition of a Poisson process, this implies that the probability that this vacancy sends $n$ offers in $dt$ is
\[
\Pr[N(dt) = n] = e^{-adt} \frac{(adt)^n}{n!}
\]  
(A1)

Let us use the following:

**Definition 1:** The function \(f(x)\) is said to be \(o(x)\) (\(f(x) = o(x)\)) if \(\lim_{x \to 0} \frac{f(x)}{x} = 0\).

Then let us state the following:

**Proposition 1:**
(i) In a small interval of time \(dt \to 0\), the probability that one job offer is sent by the finite set of the \(V\) vacancies amounts to \(V \, adt\).

(ii) In a small interval of time \(dt \to 0\), the probability that more than one job offer is sent by the finite set of the \(V\) vacancies goes to zero.

**Proof:** From (A1) one gets, using the facts that \(e^{-ax} \sim 1 - ax\) and that \(ax^2\) is \(o(x)\) according to definition 1:

\[
\lim_{dt \to 0} \Pr[N(dt) = 1] = \lim_{dt \to 0} e^{-adt} a \, dt = \lim_{dt \to 0} [a \, dt - (a \, dt)^2]
\]

\[
= \lim_{dt \to 0} \left[1 - \frac{o(a \, dt)}{a \, dt}\right] a \, dt = \lim_{dt \to 0} a \, dt \quad (A2)
\]

Using the same reasoning, one gets

\[
\lim_{dt \to 0} \Pr[N(dt) = 2] = \lim_{dt \to 0} \left[\frac{e^{-adt} (a \, dt)^2}{2!}\right] = \lim_{dt \to 0} \frac{1}{2} [(a \, dt)^2 - (a \, dt)^3]
\]

\[
= \lim_{dt \to 0} \left[\frac{o(a \, dt)}{a \, dt}\right] \frac{a \, dt}{2} = 0 \quad (A3)
\]

and, more generally, for any \(n \geq 2\)

\[
\lim_{dt \to 0} \Pr[N(dt) = n] = \lim_{dt \to 0} \left[\frac{e^{-adt} (a \, dt)^n}{n!}\right] = \lim_{dt \to 0} \frac{1}{n!} [(a \, dt)^n - (a \, dt)^{n+1}]
\]

\[
= \lim_{dt \to 0} \left[\frac{o(a \, dt)}{(a \, dt)}\right] \frac{a \, dt}{n!} = 0. \quad (A4)
\]

This shows that the probability that any vacancy sends more than one offer in a small interval of time \(dt\) goes to zero.

Let us show that this result also holds for the finite set of \(V\) vacancies. Let us denote by \(N_V(dt)\) the number of job offers sent by the \(V\) vacancies in the interval of time \(dt\). Assuming that vacancies send offers according to independent Poisson processes with parameter \(a\), Equation (A1) implies that

\[
\Pr[N_V(dt) = 1] = V\{Pr[N(dt) = 0]\}^{(V-1)} \Pr[N(dt) = 1]
\]

\[
= Ve^{-(V-1) \, a \, dt} e^{-adt} a \, dt = Ve^{-V \, adt} a \, dt
\]
accordingly, using $e^{-ax} \sim 1 - ax$, one gets

$$
\lim_{dt \to 0} \Pr[N_V(dt) = 1] = \mathcal{V} \lim_{dt \to 0} (1 - Vladt) adt = \mathcal{V} \lim_{dt \to 0} [adt - V(adt)^2]
$$

$$
= \mathcal{V} \lim_{dt \to 0} \left[ 1 - V\frac{o(adt)}{adt} \right] adt = \lim_{dt \to 0} Vladt
$$

this equation proves the first part of the proposition.

For any $n$ such that $2 \leq n \leq V$, noticing that $\lim_{dt \to 0} \Pr[N(dt) = n] = 0$, one can write

$$
\lim_{dt \to 0} \Pr[N_V(dt) = n] = C^n_V \left\{ \lim_{dt \to 0} \Pr[N(dt) = 0] \right\}^{(V - n)}
$$

$$
\times \lim_{dt \to 0} \Pr[N(dt) = 1]^n
$$

$$
= C^n_V \lim_{dt \to 0} e^{-(V-n)adt} e^{-nadt} (adt)^n
$$

$$
= C^n_V \lim_{dt \to 0} e^{-Vadt} (adt)^n
$$

(where $C^n_V = \frac{V!}{(V-n)!n!}$), which implies that

$$
\lim_{dt \to 0} \Pr[N_V(dt) = n] = C^n_V \left\{ \lim_{dt \to 0} (1 - Vladt)(adt)^n \right\}
$$

$$
= C^n_V \left\{ \lim_{dt \to 0} (adt)^n - V(adt)^{n+1} \right\}
$$

$$
= C^n_V \lim_{dt \to 0} \left[ \frac{o(adt)}{adt} \right] adt = 0.
$$

\[\blacksquare\]

A.2. The Probability of Getting a Job through Social Networks

Let us assume that an employee in the network of an unemployed individual gets a job offer. The probability $\pi$ that this unemployed worker gets the job offer is (i) the probability that none of the other $\ell - 1$ workers belonging to the network of the employee needs the job, plus (ii) the probability that the unemployed worker gets the job if a set $S \neq \emptyset$ of the other workers belonging to the network of the employee need the job. Thus, one gets formally:

$$
\pi = (1 - u)^{\ell-1} + \sum_{k=1}^{\ell-1} (1 - u)^{\ell-k-1} u^k \left[ \frac{(\ell - 1)!}{\ell! (\ell - k - 1)!} \right] \left( \frac{1}{k + 1} \right)
$$

$$
= 1 - (1 - u)\frac{\ell}{\ell u}.
$$
Each unemployed worker has $\ell$ links, each of whom has a probability $(1 - u)$ to be employed. The arrival rate of job offers on the labor market is $V_a$, and each employee gets an arriving job offer with probability $1/L$, since this offer can reach an employee but also an unemployed worker. Moreover, this worker can also get a job offer directly through formal methods. Thus, the arrival rate of job offers for the unemployed worker with $\ell$ links is

$$\frac{V_a}{L} \cdot \pi \cdot \frac{\ell(1-u)}{u} = va \left\{ \frac{(1-u) - (1-u)^{\ell+1}}{u} + 1 \right\}.$$

**References**


