Planned Contrast ANOVA

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Introduction to Planned Contrast ANOVA

• In a typical ANOVA, we are sometimes encumbered by the fact that we must test all possible mean comparisons.
• It may be that we would rather test some specific hypotheses.
• In order to do this, we need can use linear regression techniques (although we have not covered this yet) to compare the differences between pairs of means.
• These contrasts can only compare two means at once, but we can combine multiple means from different levels to compute mean pair tests (e.g., students in Programs 1 and 2 vs. students in Programs 3 and 4)
Hypothesis Test(s) in Planned Contrast ANOVA

- Recall that the null hypothesis for a one-way ANOVA can be written as:

\[ H_0 : \overline{X}_1 = \overline{X}_2 = \overline{X}_3 \]

- For the planned contrast ANOVA, we can test any specific contrast that we wish to test, but may only test a total of \( K - 1 \) contrasts.

- In the case of a one-way ANOVA with three levels, we could test a total of \( K - 1 = 2 \) contrasts.

- These hypotheses could potentially be:

\[ H_{0:C1} : \overline{X}_{P1} = \overline{X}_{P2} \]
\[ H_{0:C2} : \overline{X}_{P1andP2} = \overline{X}_{P3} \]
Making Contrasts

- In creating the contrasts, we assign new values to the groups that we want to test with each contrast.
- In the case of the study in the previous slide, we would setup the contrasts as follows:

<table>
<thead>
<tr>
<th>Program</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>
Simple Heuristic Example in R

```r
> pcdata <- data.frame(program = factor(rep(1:3, each = 2)),
+    score = c(9, 10, 11, 10, 12, 13))
> pcdata

   program score
1        1    9
2        1   10
3        2   11
4        2   10
5        3   12
6        3   13

> anova(aov(score ~ program, pcdata))

Analysis of Variance Table
Response: score
Df  Sum Sq Mean Sq F value  Pr(>F)
program    2 9.3333 4.6667 9.3333 0.05152
Residuals  3 1.5000 0.5000
```
Setting Up the Contrasts

```r
> pcdata$c1 <- rep(c(1, -1, 0), each = 2)
> pcdata$c2 <- rep(c(1, 1, -2), each = 2)
> pcdata

program score c1 c2
1 1 9 1 1
2 1 10 1 1
3 2 11 -1 1
4 2 10 -1 1
5 3 12 0 -2
6 3 13 0 -2
```
Running the PC ANOVA

```R
> anova(lm(score ~ c1 + c2, pcdata))

Analysis of Variance Table
Response: score
   Df Sum Sq Mean Sq F value Pr(>F)
   c1  1 1.0000 1.0000  2.000 0.25222
   c2  1 8.3333 8.3333 16.667 0.02655
Residuals 3 1.5000 0.5000

• Note that for this study, the two SS for C1 and C2 exactly equal the $SS_B$ in the one-way ANOVA.

• This is because the two contrasts are orthogonal.

> cor(pcdata$c1, pcdata$c2)

[1] 0
```
Examples of Orthogonal Contrasts

- In the past example, we saw that the contrasts were perfectly uncorrelated, which means that they perfectly carve up the $SS_B$ into equal shares.

- Here are some other examples for $K = 4$.

<table>
<thead>
<tr>
<th>Program</th>
<th>Ex 1</th>
<th>Ex 2</th>
<th>Ex 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
<td>C3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>-3</td>
</tr>
</tbody>
</table>
An Example with Non-Orthogonal Contrasts

```r
> pcdata$c3 <- rep(c(1, 0, -1), each = 2)
> anova(lm(score ~ c1 + c3, pcdata))

Analysis of Variance Table
Response: score
   Df Sum Sq Mean Sq F value Pr(>F)
c1   1 1.0000 1.0000  2.000  0.25222
c3   1 8.3333 8.3333 16.667  0.02655
Residuals 3 1.5000 0.5000

> anova(lm(score ~ c3 + c1, pcdata))

Analysis of Variance Table
Response: score
   Df Sum Sq Mean Sq  F value   Pr(>F)
c3   1 9.0000 9.0000 18.0000 0.023985
  c1   1 0.3333 0.3333  0.6667 0.474022
Residuals 3 1.5000 0.5000
```
In-Class Homework

- Get the dataset at http://faculty.smu.edu/kyler/courses/7311/pdata2.txt
- Look at each of the means and try and determine the best planned contrasts to run.
- Run the PC ANOVA after examining the means and SDs. Also, you might want to look at the boxplots for the study.
In-Class Homework 2

- Get the dataset at http://faculty.smu.edu/kyler/courses/7311/padata3.txt
- Look at each of the means and try and determine the best planned contrasts to run.
- Run the PC ANOVA after examining the means and SDs. Also, you might want to look at the boxplots for the study.